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Why Teach Mathematics?¹

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DURING the past decade, certain areas in the curricula of the common school have been relatively static while other areas have been modified, in some cases, to the extreme. In the first few grades of the elementary school one finds numerous changes that have taken place and are still taking place. Many of these changes are indeed significant, particularly in arithmetic. In the middle elementary grades one finds a diminishing of the number and importance of the changes as compared with those of the lower grades. Again in the junior high school much has been done to adapt materials to the needs of the age. In mathematics adjustments have been made particularly at this level. Yet in spite of the fact that the mathematics of the junior high school has been modified to meet the needs of general education, one finds the attacks continuing, especially those directed at tenth grade mathematics which has not been altered as much as the ninth grade program.

With all these attacks on the subject, the question of the place of mathematics

in the high school program should cause thoughtful teachers and administrators to ponder its values. In order to assist in this thoughtful consideration, some of the contributions of mathematics to modern civilization are presented below.

Ever since the medieval ages when scientific man borrowed the assumption of a rational universe from the Christians of the Dark Ages, he has found that mathematics is an essential language for the description of the relationships found in this universe which he assumed to be rational. For example, Galileo, in his study of falling bodies found it essential to have a method for expressing the relationship between time and distance. Mathematical symbolism was the answer, although at the time of Galileo this symbolism had not been perfected and hence his studies were to that extent impeded. The work of science in the medieval ages brought mathematics to the front because man saw how essential it was for the further study of the world about him.

The importance of this assumption of the rationality of the universe cannot be over-emphasized, since, after the scientists of the medieval and early modern age had shown how this basic assumption together with mathematics had simplified the explanation of the physical phenomena, men like Rousseau generalized this concept to include the moral laws of mankind under this same principle of the rationality of man. Previously, political

¹ This is a portion of a report by a committee on the mathematics situation in Iowa Secondary Schools. The Committee consists of: R. L. Burch, Supt. Hudson Public Schools, Hudson, Iowa; Mary Gibbs, formerly teacher of Mathematics at Vinton, Iowa; Dr. Ruth Lane, formerly teacher of Mathematics at the University High School, Iowa City, Iowa, now at Marysville Teachers College, Marysville, Missouri; and Dr. H. Van Engen, Chairman, Head of the Department of Mathematics, Iowa State Teachers College, Cedar Falls, Iowa.

institutions and moral laws had been considered as having been established by the power of God. Thus ended "The divine right of kings." Hence, mathematics had an important part in directing the thought of mankind along the lines which men of democratic nations of today are thinking. Indeed, mathematics in the sixteenth, seventeenth, and eighteenth centuries through Descartes, Kent, Hobbes, Bacon and others directly influenced the thinking of that day as it has never done (directly) since.

Space will permit giving but one more illustration of how the sciences, and mathematics in particular, have molded the thinking of the common man.

Newton in his work on gravitation postulated absolute motion and absolute time. Under these assumptions he found that the language of mathematics gave a better picture of the universe than did any other group of postulates which have been set forth up to his time. This method of interpreting the universe was in tune with the thinking of an age which thought in terms of absolute right, absolute justice, absolute wrong and a multitude of other absolutes. A few hundred years later one finds the denial of such concepts as absolute motion and absolute time by the mathematician and physicist opening the door to the denial of some of the absolutes in our moral and religious codes thereby bringing about a social concept which holds that acts of man are right or wrong only when considered relative to the common good of the individual and the group.

Here then are at least two instances in which the thought of the mathematician and scientist have influenced the whole trend of thought of the human race as regards their whole philosophy of life and their picture of the world.

The part that mathematics has played in molding the mind of man is discussed by Shaw² as follows:

² Shaw, Charles G. *Trends of Civilization and Culture*. American Book Company, New York. 1932, p. 274.

As we have seen, the beginning of modernity appeared in printing, painting, and voyaging, but it was not the book, the picture, or the map that was to represent the modern mind. The modern mind was to express itself by a formula. Hence it would hardly be amiss to state that modernity, with all the versatility of its mind and variety of its ramifications, was made by mathematics, the application of mathematics to nature and human existence.

Even though mathematics teachers have, in general, not taken advantage of the opportunities to acquaint the pupils with the part that mathematics and the sciences have played in the intellectual development of mankind, this objective remains highly worth while. Certainly the secondary schools must attempt to acquaint the laymen of tomorrow with the broad political and intellectual trends of yesterday and today. This must be true because education must connect man with man. It must connect the man of today with the man of today as well as with the man of yesterday and with the man of tomorrow.

Rugg³ has expressed this idea very clearly:

But we shall combine them (the chief political movements in European History) with a brief reference to the related geography, economic, intellectual and aesthetic movements and factors to remind the curriculum-designer that they all should be integrated into a cultural history.

Among these factors which should be integrated, Rugg⁴ mentions:

Advances in measurement, mathematics, and sciences. The astonishing movement of original invention that led in the 1500's and thereafter to the construction and use of physical measuring instruments. The perfection of algebra, geometry, calculus, and other forms of higher mathematics, the evolution of the sciences and the systematic development of the scientific method of inquiry, of the powerful role of these as instruments for increasing the precision of thinking and of their relation to the new moods that seized man's minds, of these things we must say more.

If the study of mankind is a legitimate objective for the secondary schools then

³ Rugg, Harold E. *Democracy and the Curriculum*. Third Yearbook of the John Dewey Society. D. Appleton-Century Company, New York, 1939, p. 52.

⁴ Rugg, Harold E. *Ibid*.

how can one omit areas which have been of major interest to mankind throughout the ages? As was said before, one of our objectives was to connect man with man, to show what man has done and what man can do. If this is granted, man's fight to conquer the problem of measurement, his efforts to develop a language which would enable him to adequately organize the quantitative aspects of his daily life and of the physical universe about him, these all should have a part in the curriculum of today. As Rugg said in the above quotation, "... of these things we must say more."

The Educational Policies Commission⁵ expresses their point of view:

... the free man has basic knowledge of the history of mankind... with his creation of language and number... and his search for truth, justice and beauty, his creation of art, religion, science, and philosophy. . . .

The fact that mathematics has been found to be a necessary language for the description of the changes and relationships that occur in the universe is one of the big reasons for including it in secondary education. It has not only been found to be a necessary language in the past, as we have pointed out, but it is essential for living in the technical world of today. All about us is a world of change and a world of relationships, both quantitative and qualitative. To adequately understand these changes which occur in the physical universe one must have a command of the basic symbols and ideas necessary for expressing these ideas clearly. This language which once was the tool of only the scientist, today is the language of a large group of people because its characteristics make it indispensable in many lines of work. The language of the machinist of today involves ideas of measurement, formulas, equations and the relationship between operations. The same can be said for the

chemist, physicist, many biologists, meteorologists, aviators, radio operators, electricians and others without end. In one aspect or another these people have need of the language of mathematics developed in the past thousand years. They have need of its methods of thought and its clarity in expressing relationships.

The very fact that mathematics furnishes a language and a method of thinking about quantities needed in many vocations and avocations would be reason enough for including mathematics in a general education program. However, there is yet another point which cannot be lost sight of when considering the values of the subject.

Professor Bryson⁶ says:

We still must decide what questions are to be referred to technically trained servants of the people and what can only be decided by the responsible thinking of those concerned. The best way to make such choices, which become weightier and more crucial as our civilization develops, is to know how the experts think, to sympathize with their habits and to use, when we call them in, the same clear objective thought that we expect them to use in carrying out our orders. And for what we still must decide ourselves, we need to imitate the trained mind as well as we can.

This is an extremely important point. A society cannot stand which consists of a collection of specialized groups with no common methods of thinking and groups not understanding the methods of procedure or problems or general objectives of other groups. Basically, unity of culture and unity of purpose are based upon common understanding, common backgrounds, common interests, common purposes and mutual sympathy. In the technical society in existence today the laymen, man and woman, must have some acquaintance with the methods of the specialist and the needs of the specialist. If this understanding does not exist, how can we suppose that the layman will continue to support the specialist in all of his needs? If our boys and girls are taught that mathematics is necessary only for the en-

⁵ Educational Policies Commission. *The Education of Free Men in an American Democracy*. National Education of the U. S. and American Association of School Administrators, Washington, D. C. 1941.

⁶ Bryson, Lyman. *The New Prometheus*, Pi Delta Kappa address, The Macmillan Company. New York. 1941, pp. 45-46.

gineer and the few who become mathematicians, when they become adults will they be willing to support institutions which support mathematics? Is it possible that a utilitarian philosophy of education, which pays premiums for material usefulness only, will become so firmly established in our system and will so misguide our citizens that future generations will refuse to support not only mathematics but also many other areas of learning on the false premise that these learnings are not useful? It seems that the educators might well ponder the values mentioned in the previous quotation from *The New Prometheus*.

The historian Lynn Thorndike⁷ raises this question in the following manner:

Can we have only a caste of intellectuals, as in China and India? Will the popular demands, vulgar tastes, and utilitarian attitude lower everything to its own level and swamp civilization? Or is civilization now unfolding in more varied flower than ever before with more individuals of high rank in each field and with an ever-increasing public following which is able to appreciate their work?

It is also possible to take a long time view of the needs of society insofar as mathematics is concerned. With the present emphasis on the technical lines of work more and more professions are being touched by the necessity for the use of the mathematical language. The world of fifty years from now will certainly be more technical than the world of today. Accompanying technical aspects, there is also the need for the language of mathematics. Past generations have found the need for mathematics ever increasing. Is it not possible that the future generation will find even more use for mathematics? What reasons do we have to believe that this ever-increasing need should suddenly have ceased in the decade following 1930 as some people seem to believe? In the 14th and 15th centuries, it was not thought necessary that the layman should know the number system that every second grade pupil of today must know. It was

⁷ Thorndike, Lynn. *Short History of Civilization*, F. S. Crofts and Company. New York. 1926, pp. 548-549.

absolutely inconceivable four hundred years ago that the layman could ever learn how to multiply and divide, and yet today our ten and twelve year old children know how to do the impossible of four hundred years ago. Are we sure that the crystal-gazing which tells us mathematics is not necessary is at all reliable?

This brings us very close to the argument that mathematics is too hard for the average student of today. As an answer, may we quote from a publication of the Educational Policies Commission. Dr. John K. Norton⁸ says:

Denial of educational opportunity, based either on assumed lack of native capacity or diligence and effort, needs to be critically examined. Some have always emphasized the innate shortcomings of the masses of mankind. Reading, writing, and arithmetical computations were once the basis of occupational preferment on the part of a small fraction of the population. It was thought that only the intellectually elite could absorb training in these skills. Now practically all learn to read, write, and cipher, and make daily use of these important tools.

Will the idea of the balance of an equation, the formula, positive and negative numbers, a real understanding of percentage, the table and graph be important understandings for the world of tomorrow? Will a technical civilization which, as Dr. Norton has pointed out, demand that we read, write and compute (tasks thought impossible for the layman of a few years ago) also demand measurement, the formula and the equation as it becomes more technical? H. G. Wells⁹ has an answer to this latter question. He says:

The new mathematics is a sort of supplement to language, affording a means of thought, about form, and quantity and a means of expression more exact, compact, and ready, than ordinary language. . . . The time may not be remote when it will be understood that for complete initiation as an efficient citizen of the new complex world-wide states that are now developing, it is necessary to be able to compute,

⁸ Norton, John K. *Education and Economic Well Being in American Democracy*, National Education Association and the American Association of School Administrators. Washington, D. C. 1940, pp. 123.

⁹ Wells, H. G. *Mankind in the Making*, Charles Scribner's Sons, New York, 1918, p. 192.

to think in averages maxima and minima as it is now understood to be necessary to read and to write.

The above are arguments which deserve thoughtful consideration. No extended discussion need be entered into to impress on the informed individual that in times of crisis such as this, the demand is even greater for laymen who are informed as to the language of the technician. In our civilization, it is imperative in times of great national effort that men and women have a basic understanding of the more elementary techniques of the specialist. In times of peace we need these elementary techniques to enable civilization to con-

tinue its progress to newer heights. They should be in the secondary curricula because as P. W. Bridgeman¹⁰ says:

It may, however, well require generations of intensive education before the rational principles of thought which are necessary for dealing with the simple situation of physics are intuitively grasped and instinctively applied to the complex situation of social life.

The elementary techniques of mathematics and science must not disappear from our public schools. Certainly future generations will wish to apply them to complex social situations.

¹⁰ Bridgeman, P. W. *The Nature of Physical Theory*. Princeton University Press. 1936, p. 4

The General Federation of Women's Clubs Metric Resolution

At the Annual Convention of the General Federation of Women's Clubs held April 25-28, 1944 in St. Louis, the following resolution was introduced and adopted unanimously by the delegates. This organization represents 16,500 clubs and 2,500,000 individual members.

WHEREAS, the irregular, numerous, unwieldy, and complicated units of weights and measures used in the United States and Great Britain are a hindrance to the teaching of arithmetic, every day commercial transactions, and world trade, and

WHEREAS, the metric system of weights and measures has only three units; meter, liter, and gram, interrelated and decimally divided like our dollar, and

WHEREAS, the metric system is now used in the United States in science, some factories, jewelry and optical industries, all electrical and radio measurements, athletic events, some hospitals and government departments, and especially at present in the manufacture of ammunition, and

WHEREAS, the Council on Pharmacy and Chemistry of the American Medical Association has recently decided that henceforth it will use only the metric system, and

WHEREAS, the gradual introduction of the metric system in this country (exactly as it has been introduced in 55 other countries) is feasible, and

WHEREAS, the full adoption of the metric system by the United States would be of great benefit to this country in post-war reconstruction, in promoting international commercial relations, particularly with the countries of Latin America, Continental Europe and Asia, therefore be it

RESOLVED, that the General Federation of Women's Clubs in Convention assembled, April, 1944, endorses legislation in Congress for the nation-wide adoption of the metric system of weights and measures.

The foregoing resolution was drawn up and presented by Eleonore F. Hahn (Mrs. Otto Hahn) Member of the Board of Directors of The General Federation of Women's Clubs.

Do I Have to Take Mathematics?

By SISTER NOEL MARIE

College of Saint Rose, Albany, N. Y.

THIS too frequently repeated question puts mathematics in the same category as spinach. Why spinach? Because of the young American who used to wail (before the days of Pop-Eye), "Do I have to eat spinach?" I wonder if the answer for both questions is, "Yes, it is good for you." Judging by the number of high school graduates who are making application to enter college and who have had a minimum of mathematics in secondary school, many principals and guidance directors do not think mathematics at all necessary for the intellectual life of their students.

There is no necessity for expounding here on the merits of mathematical study. Teachers of mathematics know those advantages. They know them, but is it not true that sometimes they do not see the forest for the trees? So many of our pupils are merely animated computing machines. Take the problem of solving simultaneous linear equations. Textbooks often group the pairs of equations with a bracket. I have seen pupils who are at a loss how to proceed if that bracket is missing. To them, "Solve for x and y " (with the aid of a bracket) has no connection with, "Find a value of x and y that satisfies both equations," or "What are the coordinates of the point of intersection of these two lines?" In the same category fall problems that require a graphical solution. So accustomed are they to plotting functions of the form $y = ax^2 + bx + c$ that they are at a loss how to begin plotting $ax^2 - bx - c - y = 0$. These pupils do not belong, either, to the group that is incapable of learning mathematics. They are the better type pupil. Paul R. Neureiter said in the May 1944 issue of *THE MATHEMATICS TEACHER*, "... it is not uncommon for the college teacher to see a skillful operator in algebra suddenly committing a mathematical

atrocious with the blithe air born of innocence." They lack an *awareness* of what each problem implies.

What are the results of this situation, other than the momentary exasperation of the teacher? Note the number of "mathematical proofs" that have been appearing in newspapers and magazines. Advertisers, seizing on this blind spot of the average citizen, quote figures and more figures to prove that their product is the best. One cleanser "cleans 34 more bathtubs per can than any other leading cleanser." If you do not use a certain dentrifice "your chances are 8 in 10 that you will risk cavities." And it may be proved that by using a particular soap that "2 out of 3 women may have more beautiful skin in 14 days."

The group that really proves everything by means of figures is the "cause" group. They make (so-called) statistical surveys to show that they must be right because such-and-such a percentage of people think as they do. Even highly educated people accept these "findings" as true. They lack a critical discernment whenever numbers are quoted. If we are making plans to teach "social mathematics" in the post-war period, why not include a unit to mathematically condition our future citizens?

In an address delivered under the auspices of Phi Beta Kappa, December 29, 1940, and printed in the *American Scholar* Walter Lippmann made a sweeping indictment of modern education. He asserts that "what is now required in the modern educational system is not the expansion of its facilities or the specific reform of its curriculum and administration but a thorough reconsideration of its underlying assumptions and its purposes." At present John Q. Citizen is woefully lacking in an

understanding of what constitutes a mathematical proof. The awe with which he regards a teacher of mathematics is amusing until we see him completely hoodwinked by those who take advantage of his unfamiliarity with and his fear of fig-

ures. According to Carlyle, "The best education is to train rather than to stock the mind." Using his definition as a criterion, are we teachers of mathematics giving the *best* education?

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Using Geometry in Algebra

By JOHN H. WHITE

Adelphia Academy, Brooklyn, N. Y.

THE recent demand for instruction in navigation has brought out the value of scale drawing in actual computation. In many instances it is possible to secure the answer to a navigation problem by scale drawing in a few minutes whereas calculation by means of logarithms would take much longer and would involve trigonometric manipulations beyond the grasp of the average student. It is true that the answer is not accurate to more than two significant figures, but a pilot seldom requires greater accuracy than this, and to him every second counts.

There is one class of problems the solution of which is unique and of particular interest. The solution of this class of problems may be extended to many practical problems leading to the same type of equation. A pilot is interested in getting back to his base as well as in reaching his destination. He wishes to find out how far from his base he may fly and still return within an hour. This distance is called his *radius of action*. The algebraic solution would be:

Let x be his rate out in miles per hour, y his rate back in miles per hour, and r his radius of action in miles. Since the time out plus the time back equals one hour, the equation would be:

$$\frac{r}{x} + \frac{r}{y} = 1 \quad \text{or} \quad \frac{1}{r} = \frac{1}{x} + \frac{1}{y}$$

The graphic solution is so simple that it can be understood by any student in a very short time. Take any working line and on it select two points, A and B , at random. Erect perpendiculars to this line at these points, and mark off AC equal to the rate out and BD equal to the rate back. If we now draw CB and AD they will intersect at a point E , the perpendicular distance to which is the radius of action (Fig. 1).

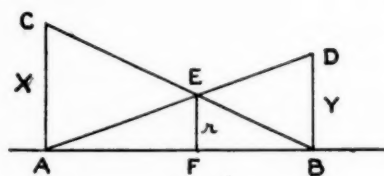


Fig. 1

The proof of this construction is a nice problem in geometry. Since $\triangle ABC$ and $\triangle BEF$ are similar,

$$\frac{r}{x} = \frac{m}{m+n},$$

and since $\triangle ADB$ and $\triangle AEF$ are similar,

$$\frac{r}{y} = \frac{n}{m+n}.$$

Adding these two expressions we have,

$$\frac{r}{x} + \frac{r}{y} = 1$$

which is exactly the same relationship we obtained algebraically.

Inasmuch as this construction applies equally well to any problem whose solution leads to a simple reciprocal equation I have outlined several practical illustrations of this method.

Ex. 1. If we have a spherical concave mirror at M (Fig. 2), a source of light at S , and the focus at F , the image appears at

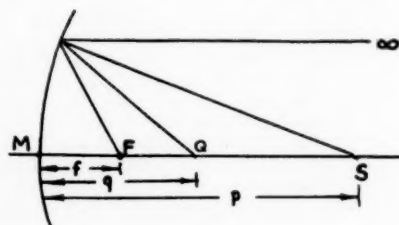


Fig. 2

Q. If distances are all measured from M , the following law has been shown to hold true.

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}.$$

Incidentally, this is an excellent exercise in similar triangles. The graphic solution of this equation is shown in Fig. 3.

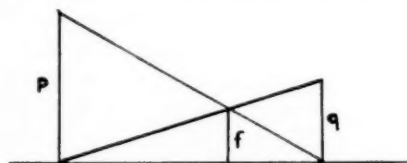


FIG. 3

Ex. 2. The resistance equivalent to several resistances in parallel, if there are no emfs in the various branches, is given by the equation:

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} \quad (\text{Fig. 4}).$$

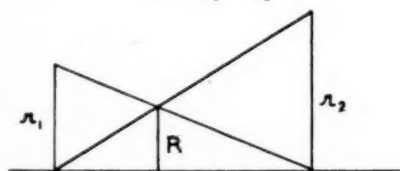


FIG. 4

Ex. 3. The conductance equivalent to two conductances in series when there are no emfs in the path, is given by:

$$\frac{1}{G} = \frac{1}{g_1} + \frac{1}{g_2}.$$

Ex. 4. The capacitance, equivalent to several condensers in series, is given by:

$$\frac{1}{c} = \frac{1}{c_1} + \frac{1}{c_2}.$$

Ex. 5. Every algebra book contains work problems. They are usually considered the hardest type of verbal problem in that they involve reciprocals. For example:

If A can do a job in 4 days, and B can do the same job in 5 days, how long will it take them to do the job together? If x equals the time required, the algebraic equation becomes:

$$\frac{x}{4} + \frac{x}{5} = 1 \quad \text{or} \quad \frac{1}{x} = \frac{1}{4} + \frac{1}{5}.$$

Fig. 5 shows the graphic solution.

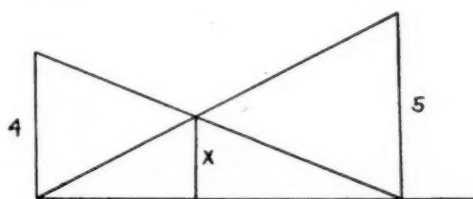


FIG. 5

Ex. 6. In each of the examples above we were restricted to two terms on the right. It often happens that we have several resistances in parallel and wish to measure their combined effect. The equation might then be:

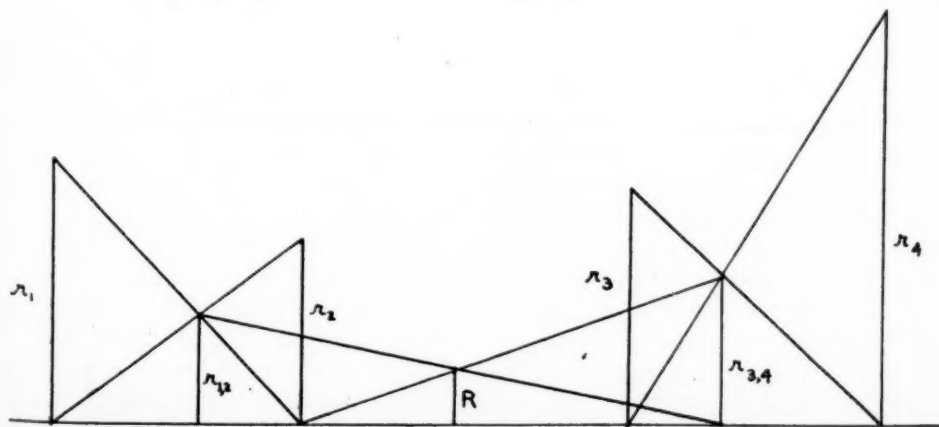


FIG. 6

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4}$$

in which case the solution would be as in Fig. 6.

Ex. 7. The graph is also interesting from the point of view of change. If y remains fixed and z increases it can be shown that x approaches y or that $1/z$ approaches 0. If z and y both increase, x increases. If z and y both decrease, x decreases. An opportunity is afforded to present variation and change in a different light and in a manner convincing to elementary students. See Figs. 7 and 8.

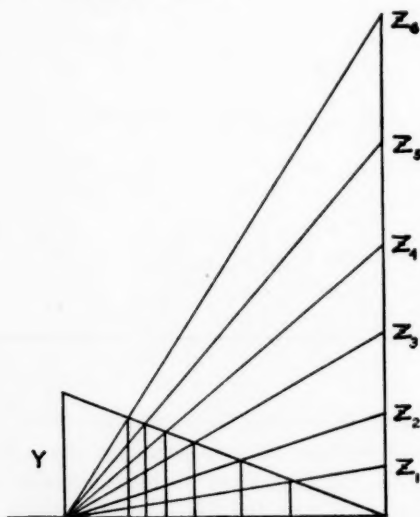


FIG. 7

Ex. 8. Up to this point we have restricted our examples to positive values. We often have an equation in the form:

$$\frac{1}{z} = \frac{1}{x} - \frac{1}{y}$$

If we agree to take values above the line as positive and those below the line as negative, there is no inconsistency in the solution. We notice that if y is greater than x , the value of z is positive as it should be (Fig. 9). If y equals x , there is no solution because the lines are parallel and do not intersect. This illustrates the principle that $1/z$ equals 0 has no solution. If y is less than x the value of z is negative as it should be.

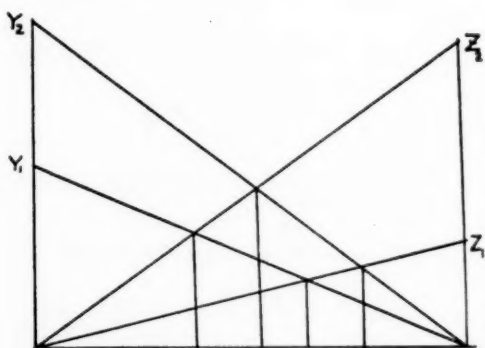


FIG. 8

Ex. 9. The focal length of a convex lens can be computed by experiment. But to

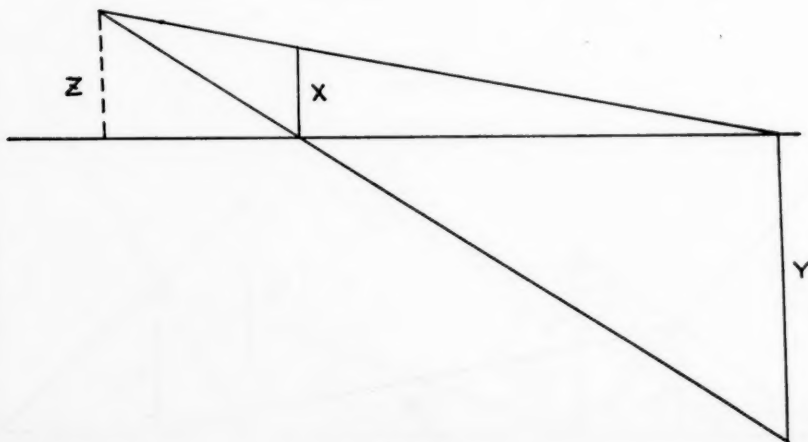


FIG. 9

find the focal length of a concave lens, it is necessary to make use of a convex lens whose focal length, f , is known. We can then measure the combined focal length, F , and compute the focal length, g , of the concave lens from the formula:

$$\frac{1}{g} = \frac{1}{F} - \frac{1}{f} \quad (\text{Fig. 10}).$$

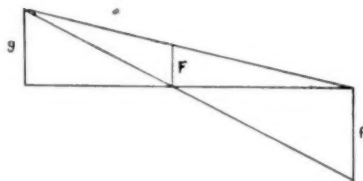


FIG. 10

As a practical measure, the closer the absolute values of the denominators are, the closer the vertical lines should be spaced in order to keep the graph on the paper.

Ex. 10. A swimming pool has a pipe which brings in the water and a pipe which drains it off. The object is to keep the water moving constantly. If the pipes are not equal and the inflow pipe requires 10 hours to fill the pool alone while the drain would empty it in 6 hours alone, how long would it take to drain the pool? (Fig. 11).

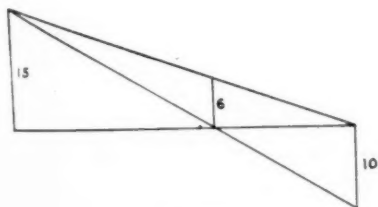


FIG. 11

Ex. 11. Every trigonometry book poses the problem of finding the height of a cliff whose base is inaccessible. Angles of elevation are read at points A and B and the distance AB is measured (Fig. 12).

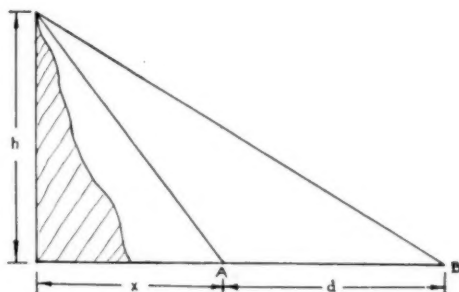


FIG. 12

$$\tan A = \frac{h}{x}, \quad \tan B = \frac{h}{x+d}.$$

Eliminating x between these two equations we have

$$x = \frac{h}{\tan A} = \frac{h}{\tan B} - d$$

or

$$\frac{1}{h/d} = \frac{1}{\tan B} - \frac{1}{\tan A}.$$

By the graphic method we may find h/d and compute h with very little effort (Fig. 13).

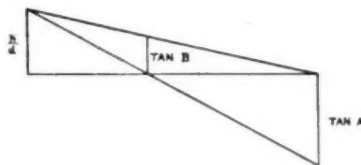


FIG. 13

Ex. 12. It is only natural to continue this process to those problems involving several terms on the right, some positive and some negative. Thus in example 10 we might have several pipes filling and several pipes draining at the same time. The solution is found graphically by combining the terms two at a time without repeating. For example the filling pipes might require x_1 and x_2 hours respectively to fill the tank alone. The drains might require y_1 and y_2 hours to drain the pool alone. The time

required to drain the pool would be given by:

$$\frac{1}{T} = \frac{1}{y_1} + \frac{1}{y_2} - \frac{1}{x_1} - \frac{1}{x_2}$$

and the solution can be formed from Fig. 14.

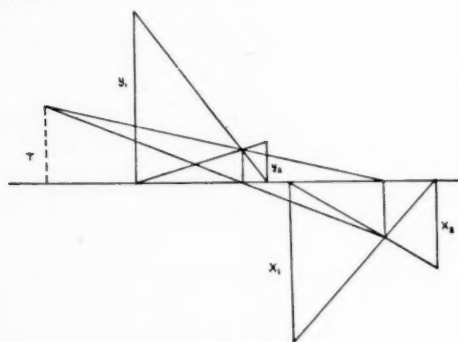


FIG. 14

It is to be noticed that it makes a difference only in sign as to whether we take the drains or the fills as negative. It is to be presumed that the student will have a fairly large sheet of graph paper ruled inches and tenths. The more involved the problem becomes the more time and effort are saved by this method.

Ex. 13. Every high school teacher is confronted at one time or another with one of those knotty problems which make the rounds and whose solution is either quite difficult or impossible. One such problem is that of the two ladders thirty and forty feet long. They are braced at opposite sides of a street and lean against windows on the opposite sides. We are not told the height of the windows or the width of the street but note that they cross at a point which is ten feet above the ground (Fig. 15). If we note that this is the same figure as that of our graphic method, we will not have much difficulty. First we reduce our scale from 40, 30, 10 to 4, 3, 1, thus simplifying the computation. Since

$$\frac{1}{10} = \frac{1}{10a} + \frac{1}{10b} \text{ or } 1 = \frac{1}{a} + \frac{1}{b} \text{ or } a = \frac{b}{b-1}$$

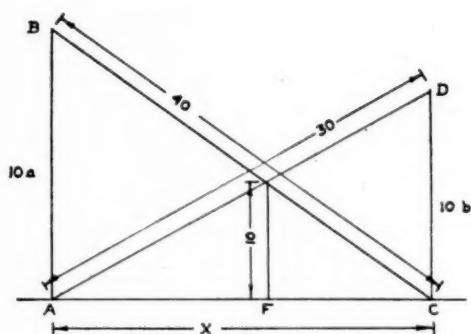


FIG. 15

By the Pythagorean relationship $16 - a^2 = x^2$ and $9 - b^2 = x^2$. By subtraction we have $a^2 - b^2 = 7$. This can be solved simul-

taneously with $a = \frac{b}{b-1}$, giving $b^4 - 2b^3$

$+ 7b^2 - 14b + 7 = 0$. This is a quartic or bi-quadratic equation, but good approximate solutions can be obtained by the engineering method, involving division by detached coefficients. $a = 3.02$, $b = 1.49$, $x = 2.60$. If we multiply each value by 10 we have the true dimensions. It is not my purpose to explain the engineering method, but this is the only practical method of obtaining a quick solution and may be taught to any class in high school algebra. It will be received with more enthusiasm and be far more valuable than much of the material which now exists in such courses.

The Seventeenth Yearbook of The Na-

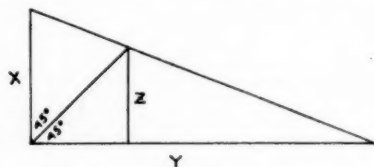


FIG. 16

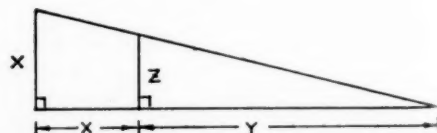


FIG. 17

practical. Some others are outlined in Figs. 16, 17, 18. Figure 18 represents a very interesting application in navigation. If BC represents the ground speed out, and AB the ground speed back, and BD the speed of the wind, then BF is the radius of action. DC equals AD and is the air speed.

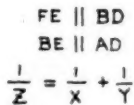


FIG. 18

Film Available!

THE FILM "Scientists For Tomorrow" is available for showing, without charge, through the Motion Picture and Speakers Bureau of the Westinghouse Electric & Manufacturing Company, 306 Fourth Avenue, Pittsburgh, Pa.

Relational Thinking as a Criterion for Success in College Mathematics¹

By J. R. MAYOR

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THE recent emphasis on evaluation is reaching college teachers and is beginning to have its effect on college teaching. Although the college or university curriculum allows considerable specialization, particularly in the last two years, an important objective of any college mathematics course could be to teach relational thinking.

In a discussion of trends in college mathematics accompanied and followed by illustrations, which indicate the role of relational thinking in college mathematics, one should find some reasons why relational thinking should be used as a criterion for success in college mathematics, and also some indication as to how it can be so used. It is hoped that this organization will provide more of interest to the secondary school teacher than a direct approach. A complete discussion of means would, of course, have to take into consideration organization of the course and conduct of each class discussion, which includes lesson planning and evaluation of achievement.

It seems in place, to start with a consideration of the relation that teachers of or the teaching of college mathematics bear to high school teachers. I hope, in answering two criticisms of college teachers, to clear up some possible misunderstandings. First, college teachers of mathematics have sometimes had a bad influence on the mathematics of the high school, and in the second place, college teachers have been guilty of placing too much blame on high school teachers for the lack of preparation of college students. In answering the second, let me say that

the college teacher of to-day is coming more to realize that, that part of the difficulty of his students which is due to faulty preparation, goes back much farther than the mathematics of the high school. In fact it goes back to the first opportunity for teaching relational thinking which was neglected.

In answer to the first criticism let me say that there is much to indicate that college teachers of mathematics in the university, in the liberal arts college, and in the teachers college are realizing the vital importance of the secondary school mathematics and of their understanding of the problems and the difficulties of the high school mathematics teacher. There is no better evidence than that provided by the Fifteenth Yearbook of the National Council, "The Place of Mathematics in Secondary Education."² This Yearbook represents the work of a joint commission on which we find some men whose first interest is the teaching of college mathematics.

As another case in point here, and as a matter of great importance to all of us in itself, I would like to call attention to the report of the War Preparedness Committee of the American Mathematical Society and the Mathematical Association of America.³ The work of this group was carried on through three sub-committees: Committee on Research, Committee on

² "The Place of Mathematics in Secondary Education," Final Report of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics. Bureau of Publications, New York, 1940.

³ Report of the War Preparedness Committee, The American Mathematical Monthly, Vol. 47, August-September, 1940, pp. 500-502. See also, *Bulletin of the American Mathematical Society*, Vol. 46, September, 1940, pp. 711-4.

¹ Presented before a pre-war meeting of the Illinois Association of the National Council of Teachers of Mathematics, at Urbana.

Preparation for Research, Committee on Education for Service.

The names of the first two sub-committees indicate their objectives. The third group was set up to strengthen "undergraduate mathematical education in our colleges to the point where it affords adequate preparation in mathematics for military and naval services of any nature."

One of the three resolutions, recommended by the Committee and adopted by the Council of the Society and the Board of Governors of the Association, is:

"That all competent students in the secondary schools take the maximum amount of mathematics available in their institutions. In the case of many schools, additions to the present curriculum will be necessary in order to furnish an adequate background for the military needs of the country."

This sub-group on education for service has been set up and in the very near future a great deal of information for high school, as well as for college teachers, should be made available.

I hope that you are interested in the mathematics now taught in the colleges, the recent trends and the importance of relational thinking in all of it. I further hope that by no means the most important reason for your interest is that you are teaching students who may enroll for college mathematics. I would almost prefer that your interest come through a curiosity to know of changes and improvements, if any, since you were in college. I believe that the fine work that is being done toward improving the teaching of high school mathematics is having an effect on the teaching of college mathematics through students who come to college richer in relational thinking experiences, and also directly in the planning of college courses.

There are many angles to the problems of teaching college mathematics that I will be unable to touch upon here. There is a growing literature on the subject, par-

ticularly on the mathematics of the junior college. The preparation and interests of the student present a difficult but important problem. In the organization of mathematics courses consideration must be given first of all to the needs of the student, particularly in relation to his interests in the college. At Carbondale the sectioning of first year students is based largely on their interests. Those who plan to continue work in mathematics or science are registered in one course and all other students in a second course. This, incidentally, provides a fairly satisfactory division in relation to ability.

In this discussion, the first year of college mathematics refers to the course offered in the college or university for general education, or for preparation for more advanced work in mathematics, or for preparation for teaching mathematics. Although it may appear that the objectives for these different groups are not the same, and although in some colleges separate courses are offered for them, I believe that the discussion to follow may apply to any one or all equally well. Certainly for all three groups the importance of relational thinking cannot be minimized.

I suppose the most concrete trend in college mathematics is that of uniting the subject matter of algebra, trigonometry, analytic geometry, and even the elementary notions of the calculus in a single course usually offered under some such title as elementary mathematical analysis. In such courses, the function concept aids in the integration of subject matter usually presented in separate courses. In some texts this has been very skillfully accomplished. Naturally the amount of integration varies greatly in the different texts. There is a reference to this trend toward unified freshman mathematics in the Fifteenth Yearbook of the National Council. It is stated that out of 352 junior colleges studied, 143 offered combined courses for freshmen.⁴

⁴ "The Place of Mathematics in Secondary Education," op. cit., p. 153.

A further indication of widespread interest is found from an examination of the texts suitable for a unified course which have come to our office in the past few years. The authors of these texts are on the staffs of the following colleges and universities: Oregon State College; State Teachers College, Montclair, New Jersey; University of North Carolina; Brown University; Iowa State Teachers College; University of Chicago; New York University; George Washington University; Swarthmore College; Miami University; Michigan State College; University of Michigan; University of Washington; Texas Technological College; University of Illinois.

A few of these texts go much farther from the traditional than that which is involved in an integration of algebra, trigonometry, and analytic geometry in an attempt to provide a course for general education. The Joint Commission recognized this need in stating that "any person intelligent enough to graduate from a junior college presumably should be able to carry successfully a course that is planned to give both an insight into the nature of mathematics and an appreciation of its wide and growing importance in man's life."⁵ Experimentation with a course of this type is highly important. Furthermore such experimentation should encourage the adoption of relational thinking as a criterion for success.

There are other trends in teaching college mathematics, though neither so marked nor so widespread, that should make the emphasis on relational thinking more natural. Outside of the introduction of the calculus as a part of a first year course, not so much change can be noted in this fundamental part of the undergraduate mathematics curriculum. I regret that I cannot point to a trend toward combining the differential and the integral calculus into a single course. In my opin-

ion such an arrangement aids relational thinking in connection with some difficult concepts.

Solid analytic geometry is now more frequently introduced in connection with plane analytic geometry. It seems to me a great pity this is not always done. A similar opinion applies to plane and space concepts in projective geometry. Projective geometry can contribute much to the prospective teacher and to all other mathematics majors with the possible exception of those who are taking their major for use in another science. To indicate that relational thinking is important in projective geometry, one need only call attention to the opportunities presented in the application of the principle of duality.

The content and organization of undergraduate courses in mathematics have been little changed to keep astride of the most recent advances in mathematics. No more forceful criticism of this has been given than that in a paper by Marie J. Weiss.⁶ Dr. Weiss makes a plea for the inclusion in the algebra of the senior college of concepts which are of interest to the working mathematicians of our time. She points out that "these concepts if well understood will give the prospective teacher and general student an introduction to the domain of algebra and an understanding of a mathematical system," which to use her words provide, "surprisingly enough, the scientist some indispensable tools for advanced work." She mentions among other topics: the calculus of matrices, the elementary properties of groups, and number rings and number fields.

Perhaps partly due to the recommendations for preparation of secondary school teachers found in the 1935 report of the Commission on the Training and the Utilization of Advanced Students of Mathematics, appointed by the Mathematical Association, one finds the undergraduate

⁵ "The Place of Mathematics in Secondary Education," op. cit., p. 155.

⁶ Weiss, Marie J. "Algebra for the Undergraduate," *The American Mathematical Monthly*, Vol. 46, 1939, pp. 635-642.

mathematics major, whether or not in preparation for teaching, more apt to include financial mathematics and statistics than was true a few years ago.⁷ Also most universities fulfill in some manner the recommendation of this Commission for "either directed reading or a formal course in the history of mathematics and the fundamental concepts of mathematics." This recommendation is included in the Fifteenth Yearbook.

Now we all know well enough that putting materials together under the same text cover and spending this week on solving right triangles and next on finding the zeros of polynomials or the present value of an annuity due, in itself, is not a solution of any problem of teacher or student. Some teachers of the traditional college algebra are a great deal more successful in bringing students happily through experiences in relational thinking or even, to use another popular term, "creative esthetic experiences," than other teachers with the best unified text in the world. The teacher himself must be conscious of the relations, and must lead the students to see the possibilities of those presented and to discover many new ones. Personally I believe that this is easier to do and more frequently done, in an elementary analysis course where an attempt is made to show the relations through emphasis upon them.

A very simple case might be made of the situation in which a student has no particular difficulty in handling a combination of fractions such as

$$\frac{1}{x} + \frac{x}{x-1} \text{ but is confused by } \frac{1}{\sin x} + \frac{\sin x}{\cos x}.$$

Presenting these together should not only save time but, also I believe, should give the pupil a much clearer notion that the operations of arithmetic and algebra are used in combining trigonometric functions of x in the same manner as other functions of

x , an understanding which has much to contribute to success.

Or consider the class which had been assigned exercises in plotting in polar coordinates. One of the boys, John Jones, was successful in plotting the entire list and on the following day he could present an accurate sketch and a correct table of values for any of the curves. Bill Smith, on the other hand, had not completed all of his graphs. Perhaps this was because he had tried to determine for himself, and had largely succeeded, why the graph of $\rho = \sin 3\theta$ is a three-leafed rose while the graph of $\rho = \sin 2\theta$ is a four-leafed rose. Further he was puzzled by the equations of the type $\rho = a + b \cos \theta$ and had attempted to formulate some conclusions about the relation of the relative sizes of a and b to the curves. During the class period Smith asked some questions indicating his thinking in terms of relations that greatly stimulated the class discussion. Now not all of us would agree that Jones, the first man, is the one who would achieve the greater success. In such a case is not relational thinking a better criterion for success?

Let me call your attention to an address by the late E. R. Hedrick presented for the Slaughter Memorial volume of the Monthly.⁸ Hedrick emphasizes the use of functional thinking from the first application of the simplest multiplication combinations up through the theory of the functions of a complex variable. I agree that in analytic geometry it is of primary importance "to know how to indicate graphically the behavior of relations which are given by equations," and that considerable attention must by all means be given to relations between quantities expressed by such simple equations as $y = x^2$, $xy = 4$, $y = 3^x$, and $y = \sin x$. I hope that I don't disagree too much with him when I also value the consideration of the general equation of second degree and the properties common

⁸ E. R. Hedrick, "The Function Concept in Elementary Teaching and in Advanced Mathematics," *The American Mathematical Monthly*, Vol. 45, 1938, pp. 448-455.

⁷ "Report on the Training of Teachers of Mathematics," *The American Mathematical Monthly*, Vol. 42, 1935, pp. 263-277.

to all of the conics. I believe that relational thinking involved in the study of the general second degree equation can contribute an understanding, essential in mathematics, in its application and appreciation.

Analytic geometry is so completely a study of relations that the student who has not acquired good habits of relational thinking finds this work extremely difficult. It is at this point that quite a considerable number turn to other fields of study to which they can adjust their kind of thinking more easily.

The study of the graph of the equation $y = e^x \sin x$, is not commonly found in courses in analytic geometry, nor even in the calculus. However, this particular equation provides an unusual opportunity to teach (and to determine the success of past teaching) certain relations in connection with the two important functions involved and the effect of the product of two functions. Incidentally, interesting physical relations involving the application of these functions may also be very profitably brought into the discussion.

In a study of applications of integration in the calculus, the treatment of area under a curve is sometimes followed by the discussion of work done by a variable force. Along with the "work" problems, an exercise may be included in which students are asked to construct figures and show that the work done by a variable force can be represented graphically as the area under a curve whose ordinates represent the force. Frequently students can, by following a model, do all of the problems assigned on this topic except this one. In my opinion the student who can present a good discussion of this particular exercise shows that he understands the important relations and concepts involved in these simpler applications of the integral calculus and such an accomplishment provides a real test of success.

A relation, simple to state, $e^{i\pi} + 1 = 0$, is encountered in the second year of college mathematics. It has been the source of

much inspiration to some very great men. David Eugene Smith has made the statement: "The formula, $e^{i\pi} = -1$, expresses a world of thought, of truth, of poetry and of religious spirit."⁹ This relation is an extremely compact example which suggests that relational thinking could provide a criterion for success in college mathematics.

I do not want to leave this phase of the discussion without turning to a part of college mathematics, referred to above as more recently popular—mathematics of finance. From the first, accurate thinking is required, as for example, in understanding the relation between simple interest and simple discount.

Later on, the student must certainly have an understanding of time and quantity relations if he solves an equation of value; or determines the monthly payment, for principal and interest, due on a loan of \$5000 to build a house; or finds the difference in the annual premium on a 20-year term and a 20-year endowment insurance policy for a person age 18.

In college mathematics, too, to measure success we must emphasize concepts more and mechanics less. The seven concepts "basic to problem-solving, crucial in democracy, and pervasive in mathematics," which are so adequately discussed in "Mathematics in General Education," the report of the Commission on Secondary School Curriculum of the Progressive Education Association, must also be taught through college mathematics.¹⁰

Fortunate indeed is the mathematics teacher who has read the two excellent re-

⁹ This quotation is a part of one sentence from a statement by David Eugene Smith, headed the Science Venerable. It is printed in a very attractive form on p. 110 of *Scripta Mathematica* for April, 1938. The statement is written in the characters used in a parchment document dated 1139.

¹⁰ "Mathematics in General Education." Report of the Committee on the Function of Mathematics in General Education, for the Commission on Secondary School Curriculum, Progressive Education Association, D. Appleton-Century, Inc. New York, 1940.

ports of 1940, and several other fine books, by no means the least of which is *Mathematics and the Imagination* by Kasner and Newman.¹¹ My sincere sympathy goes to the reader insufficiently trained in mathematics and relational thinking to enjoy that recent book.

A possible interpretation for this topic assigned to me could have been one devoted entirely to methods of constructing tests on the college level by means of which success in college mathematics could be judged upon ability to do relational thinking. You are probably well aware that not many such tests have been prepared for publication. Excellent samples of such tests can be found, however, for secondary school mathematics, for example in the Ninth Yearbook of the Council¹² or in the PEA report "Mathematics in General Education."¹³ Here

¹¹ Kasner, Edward and Newman, James. *Mathematics and the Imagination*, Simon and Schuster, New York, 1940.

¹² "Relational and Functional Thinking in Mathematics." National Council of Teachers, of Mathematics, Ninth Yearbook, Bureau of Publications, Teachers College, New York, 1934.

¹³ "Mathematics in General Education," op. cit. Chapter XIII.

again the high school is leading the college.

If the most is to be made of relational thinking in mathematics more direct attention must be given to it in the evaluation program. Could it have a better single stimulus than that provided by tests which are actually tests of mathematical relations? If proper tests in college mathematics were prepared it should not be difficult to use relational thinking, as a standard for judging success, beginning with the sectioning of students through to the final record of the teacher in the A, B, C, D, or some other form. The preparation of tests of mathematical relations is a difficult task but any teacher can improve his present output with a little attention to such an objective.

In this brief contact with evidence of increasing interest in secondary mathematics on the part of college teachers, with a discussion of recent trends in and with illustrations from college mathematics, I hope that answers to some of the questions regarding college mathematics may have been found, and with these answers a strengthened realization of the need for teaching relational thinking in mathematics on all levels.

Army Outlines Pre-Induction Needs to Schools*

"How can teachers and school administrators help to prepare boys for military service in the period prior to induction?" The War Department answers that question for schools of the nation in a new bulletin, "Essential Facts About Pre-Induction Training."

According to the bulletin, all men faced with induction into the Army need:

1. Physical fitness
2. Basic mathematical and language skills
3. Knowledge of and ability to apply scientific principles
4. Occupational skills
5. Appreciation of the cause for which we fight
6. Acquaintance with Army life and training procedures
7. Understanding of principles of health, sanitation, and first aid
8. Knowledge and skill in rifle marksmanship, military drill, and map reading.

War Department Bulletin PIT-1, "Essential Facts About Pre-Induction Training," U. S. Government Printing Office, Washington, D. C. 1944. (Prepared by the War Department in cooperation with the U. S. Office of Education.)

The Mechanism of Gravitational Force

By EMERY E. WATSON

Iowa State Teachers College, Cedar Falls, Iowa

ALTHOUGH it has been known since the time of Newton, nearly three centuries ago, that bodies attract each other according to the inverse-square law, no explanation as to how gravitation operates has been accepted. However, with our present knowledge of matter it would seem that some explanation should be possible. Keeping in mind Newton's first law of motion; namely, "Every body continues in its state of rest or uniform motion in a straight line unless compelled to alter that state by impressed force," let us consider, for the purpose of illustration, an atom within which the motion is produced by the vibration of the nucleus in whole or in part, or by the electron as it rotates on its axis or as it travels in its orbit around the nucleus. The direction of rotation or revolution of the particles within the atom is apparently unimportant since the nucleus of the atom may rotate with or in opposition to the motion of the electron as is evidenced by the position of the spectrum lines.

I. PRODUCTION OF SPHERICAL SPACE WAVES

Since the normal motion of a body in space is a straight line, Fig. 1, and the path of the motion of the electron or other



FIG. 1
Newton's First Law

generating particles is curvilinear, it follows that for each vibration, rotation or revolution within the atom two or more forces must act in opposition. Thus, to produce the curvilinear motion of the electron some central force must be exerted by the nucleus on the given rotating or revolving particle, a central force which continually changes the motion of the particle

from that expressed by Newton's first law; namely, "motion in a straight line," to motion in a closed orbit. As a result of these two opposing forces, a somewhat spherical shaped field of force lying along the path of the electron (or other revolving particle) is generated, one surface for each revolution of the electron or particle.

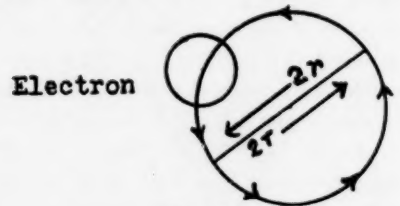


FIG. 2
The Atom

These successive, concentric, expanding, spherical shaped fields of force lie within the atom or closely surrounding it.

Once generated, these successive, spherical shaped fields of force are assumed to have a radial velocity of 186,000 miles per second, a velocity which is dependent not on their origin (i.e. on the velocity of the electron, here represented by " u "), but on the rate of propagation of space waves.

Footnote 1. The sum of the magnitudes of these radially directed components in these successive spherical fields of force is expressed in the pull which creates an equatorial bulge of thirteen miles on the earth, an even greater equatorial bulge on Jupiter, keeps the moon from falling to the earth, the innumerable satellites of Saturn from falling to the planet and determines the shape of the Milky Way and many other nebulae. On the earth the outward pull at the equator is about $1/289$ of the earth's pull of gravitation at or near the surface of the earth.

2. Every atom is moving through space with a velocity varying from zero miles to several thousand miles per second, and

has both linear and curvilinear motion. The vibration of the electron is only one source of motion within the atom. In the case of the so-called stripped atoms of the very dense stars, or in the case of the neutrons which make up the helium stars, the gravitational force is apparently not lessened.

II. THE INTENSITY OF THE EXPANDING FIELD OF FORCE VARIES AS k/d^2

Since the surface of one of these expanding spherical fields of force varies directly as the square of the distance from the atom, the intensity of the field of force must vary inversely as the square of the distance from the source of generation. Hence, if the intensity of this force is the measure of attraction between two bodies, say, mass A for mass B (or mass B for mass A), then Newton's principle; namely, "The attraction between two bodies varies inversely as the square of the distance be-

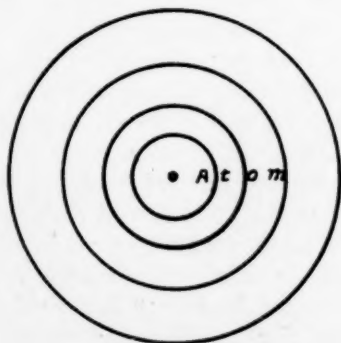


FIG. 3
Expanding Field of Force

tween them," (that is, $1 = k/d^2$) is true for any two bodies in this field of force.

III. THE ATTRACTION VARIES AS THE PRODUCT OF THE MASSES, $1 = km_1m_2$

Since each field of force originating at A will affect each atom at B which vibrates in unison or nearly in unison with the atoms at A , and each field of force originating at B , in a similar manner will affect each atom at A , the effect of A on B is

just equal to the effect of B on A . Hence the combined effect on the two masses varies directly as their product, namely, mass A times mass B .

Illustration. Let mass A consist of five atoms and mass B consist of two atoms,

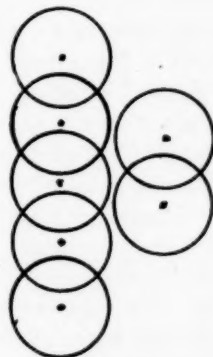


FIG. 4
Intensity Varies as the Product of the Masses

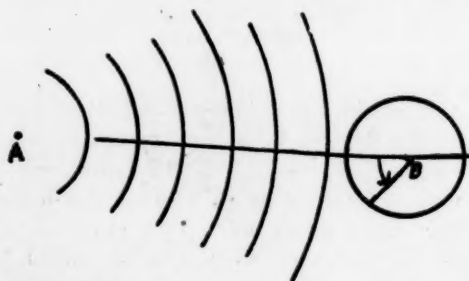
and consider that just one spherical wave in this field of force is generated by each atom. Then each of the two atoms at B may be affected by each of the five spherical waves generated by the atoms at A . Hence, if each wave effect, in which the oncoming wave is synchronized with the motion of the electron, represents a pull, then mass A will exert five pulls on each of the two atoms at B . In like manner, mass B will exert two pulls on each of the five atoms at A . That is, in each instance the pull of A on B is just equal to the pull of B on A . In general, if one mass is represented by m_1 and the other by m_2 , then the force between the masses can be represented by km_1m_2 . That is, the attraction between the masses is proportional to the product of the masses. Hence, from II and III it follows that the combined effect of these fields of force, carried as space waves from one body to another, varies directly as the product of the masses and inversely as the square of the distance between them. That is $1 = km_1m_2/d^2$, which is Newton's law of "universal gravitation."

IV. THE MOTION IN ATOM A IS TRANSFERRED BY SPHERICAL FIELDS OF FORCE TO ATOM B

Two factors enter into the transfer of energy from mass A to mass B.

- The creation of an expanding field of force.
- The synchronization of this oncoming field of force from atom A with the motion in atom B, and conversely.

Illustration. If one of two tuning forks having the same pitch is set in vibration, the other, although at a considerable distance, will vibrate in unison with the first. In the transfer of force from mass A to mass B there is a fairly close analogy between it and the transfer of sound waves from one tuning fork to another having



Successive Wave Fronts from A

FIG. 5
Orbital Motion of the Electron at B

the same pitch. In either case, the linear velocity of the tuning fork or the orbital velocity of the electron here denoted by " u " may be much less than the velocity of the particular wave at any intermediate point between A and B. However, a fairly close synchronization of the S.H.M. at A and B is requisite.

If the electron by its orbital motion creates a field of force within or surrounding an atom, then the linear distance, considered as being traversed along any diameter of the orbit at an average velocity " v " (not space velocity) is to the corresponding orbital distance traversed by the electron at a velocity " u " (not space

velocity) as $4\pi r$ is to $2\pi r$ or as 1 is to $\frac{\pi}{2}$.

It is the radial (linear) velocity rather than the orbital velocity of the electron at A that is transferred by spherical space waves to the electron at B. The average of the variable velocities " v " along any diameter within the atom is less than the constant orbital velocities " u " in the ratio of 1 to $\pi/2$ or 1 to 1.57. Hence $u = \frac{\pi}{2}v$ or $u^2 = 2.46v^2$.

Since each point in this spherical field of force originating at A traverses the intervening space at a velocity of 186,000 miles per second it has the equivalent of mass. If this mass equivalent, say m , is to be absorbed into an atom at B the work ($w = 1/2 mv^2$) done on the atom at B tending to drive it away is equal to the kinetic energy of m where m is the mass equivalent of the portion of the field absorbed by, the given electron. But if this mass m having within the atom B a forward velocity " v ," is to be absorbed into the motion of the electron having an orbital velocity " u " it must be synchronized with the orbital motion of the electron at B both as to its period and as to the phase required for absorption. Since the velocity " v " is less than the velocity " u ," the mass " m " from A enters the atom B as a drag upon the orbital motion of the electron or vibrating unit. To overcome this drag, work must be done by the nucleus. But if work is done by the nucleus in absorbing this mass " m ," the mass " m " of the portion of the wave being absorbed must do an equal amount of work in the nature of a pull on the nucleus. The amount of work to be done by the nucleus is expressed by the formula, $1/2m(u^2 - v^2)$. But $1/2m(u^2 - v^2) > 1/2mv^2$ (the kinetic energy of the impact), for $1/2m[(\frac{\pi}{2}v)^2 - v^2] > 1/2mv^2$, or $1/2m(2.46v^2 - v^2) > 1/2mv^2$ since $1.46 > 1$. Hence the energy expended by the nucleus in overcoming this drag is greater than the kinetic energy of the impact. That is, the absorption of the given portion of each wave in this field of force pro-

duces, on each atom where the absorption takes place, a slight pull (or acceleration) in the direction of the oncoming wave front. The sum total of these pulls (or accelerations) in the direction given to the atoms at *B* by the successive absorptions from the field of force originating at *A*, is the so-called gravitational pull of *A* on *B*. Likewise the successive absorptions at *A* from the field of force waves originating at mass *B* is the pull of *B* on *A*. The sum total of these pulls of *A* on *B* and *B* on *A* is the total pull between the two bodies and is expressed by Newton's Universal Law; namely, $1 = km_1m_2/d^2$. That is, gravitation is purely a mechanical effect due to the spherical space waves and is common to all matter.

V. FIELDS OF FORCE ARE UNOBSTRUCTED IN THEIR MOTION THROUGH MATTER UNLESS ABSORBED INTO THE ATOMS

That an atom is mostly space is evident from the following considerations. The effective diameter of the nucleus is about 10^{-13} cm. The effective diameter of the atom is about 10^{-8} cm. Hence the distance from the nucleus of the atom to the electron must be about 100,000 times the diameter of the nucleus. From such considerations it is evident that unless some portion of these spherical waves or fields of force is absorbed into the motion of the atom to affect the electron, and only properly synchronized wave fronts can be so absorbed, their passage from one point to another point is only slightly obstructed by the presence of matter. As a result, the gravitational effect on an atom on the near side of an object or on the far side is

nearly inversely proportional to the square of the distance of the given object from the source of generation. During a total eclipse of the moon or of one of the four inner satellites of Jupiter the gravitational pull of the sun on these bodies during the period of total occultation is so slightly diminished that the decrease has not even been detected. Hence, we conclude that only a very small per cent of these waves, from the field of force, are absorbed by the earth as they pass through it and there is no such thing as a total gravitational screen.

VI. BELOW ARE LISTED POSSIBLE VERIFICATIONS OF THE ABOVE THEORY

1. During an eclipse of the moon, time about two hours or less, the sun's attraction on the moon may be decreased sufficiently to exhibit a measurable deflection of the moon's path toward the tangent.

2. During an eclipse of any one of the inner four satellites of Jupiter, the sun's attraction on a given satellite may be decreased sufficiently to show a measurable deflection toward the tangent. For these satellites, the periods of revolution are very short and the centrifugal force large. The period for Io is only one day and 18 hours.

3. The attraction between bodies of different materials, as wood, copper, gold, etc. may or may not be proportional to their masses. Hence, two masses may be compared by means of an inertia balance, and the attraction of a third body for each of these bodies compared. Bodies having unlike atoms may not meet the condition of the inertia balance.

Important Notice!

The Regional Meeting of The National Council of Teachers of Mathematics which was scheduled for February 23rd and 24th in Cleveland has been called off on account of the Government's request not to schedule such meetings.

The Mathematics Institute

By W. S. SCHLAUCH

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FOLLOWING the establishment of public school systems in the various states, the problem of increasing the efficiency and enlarging the outlook of teachers soon challenged the attention of state school authorities. Among the methods employed to raise the level of attainment in the teaching body were summer institutes for teachers, lasting from four to six weeks. From these institutes were evolved schools for training of teachers of the public schools, known as normal schools, open the year round, but attended mainly in the spring session.

The institute idea was not abandoned, however, and a one week institute for all the teachers of a county was held, usually in November. The schools were officially closed, and the teachers flocked to the county seat, where a series of lectures had been arranged by the County Superintendent, often with school work exhibits, and entertaining features.

These institutes were supposed to present approved methods of teaching, give the teachers some idea of recent literature dealing with teaching, and inspire the assembled teachers with enthusiasm for their profession. However, as the majority of the teachers were usually teachers of all subjects in one room schools, the lectures on methods were rather vague and general, and methods of teaching specific subjects were usually lacking.

With the growth of the high school movement in the latter part of the last century, the need for wider and more thorough scholarship for the teaching body, as well as a more complete analysis of the teaching process led to the establishment of teachers colleges, which could supply these needs. That teachers generally have responded to their increased opportunities and demands is shown by the census of 1940. In that year, the modal preparation

of the senior high school teachers of the United States was a four year college training and one year's graduate work, usually with the M.A. degree. The modal Junior high school teacher's training was a four year college course. For graded elementary school teachers, the modal academic preparation was graduation from high school and two years in college; usually completion of a junior college curriculum or two years of a four year college course.

For secondary school teachers, summer school courses in teachers' colleges continue to offer professional opportunities to teachers working for higher degrees. But many teachers who have received degrees at teachers colleges, and many who feel that they can not afford the expense of attending a full summer session are eager to keep abreast of movements in education, and in their specialties. For such teachers a two weeks summer institute presents a welcome opportunity.

The Institute for Teachers of Mathematics organized by Duke University under the direction of Professor W. W. Rankin has been in existence since the summer of 1941, has been attended by teachers of mathematics from eleven states, and many of them have returned in succeeding summers. So enthusiastic are the teachers in attendance over the benefits derived from the lectures, the mathematics laboratory, the group studies, and the social events and consultations with the group leaders, that it seems desirable to give some detailed exposition of the Duke plan, which may perhaps stimulate the formation of mathematics institutes elsewhere.

The first institute was held from June 17 to June 20, 1941. It was an experiment, covering only four days, but was rich in offerings. Professors of Duke, Johns Hopkins, the University of North Carolina, as well as some prominent high school teach-

ers discussed such topics as the following:

1. Some Practical Problems of Secondary Mathematics, Whose Solution Requires a Knowledge of Algebra and Geometry: in Business, in Industry and in Science.
2. Mathematical Models and Their Use in the Study of Mathematics.
3. Demonstration of the Slide Rule.
4. Field Work and Use of Instruments.
5. Traditional versus Unified Mathematics.
6. The Place of Mathematics in Secondary Education.
7. Mathematics for Students Not Going to College.

The 1943 Institute was expanded to cover eleven days, and was held from June 16 to June 26 inclusive. In this session of the institute, the shadow of war was over the land, and the application of mathematics to military affairs, including gunnery, navigation, and aviation under leadership to Professor J. M. Thomas of Duke was presented under the general heading "Mathematics for Defense."

This was a non-credit course that met throughout the institute from 8:00-9:30 a.m., and was an additional offering for those who wanted to know how mathematics functions in military affairs.

The regular institute courses, led by Duke Professors, Dr. Wm. Betz of Rochester, and prominent high school teachers again covered such live and interesting topics as:

1. A Mathematics Laboratory
2. The Teaching of Relationships
3. The Training of Teachers
4. Keeping the Curriculum up to Date
5. Conflicting Philosophies of Curriculum Development
6. The Problem of Ninth Year and Tenth Year Mathematics

Social events helped to make the institute enjoyable, and the high schools of the state were invited to make exhibits at the institute, such as models, maps, instruments, etc., and a prize of \$25 was offered to the high school making the best exhibit.

The 1943 Institute for Mathematics Teachers found itself "come of age." The work was divided into units, and teachers in attendance selected the unit with which they wished to conduct research and report findings. Here was the plan that made the participants feel that they were contributing something of value to the profession at large as well as receiving instruction. The mathematics laboratory and library under the direction of Professor Rankin was open to the Institute members during the entire session, and proved a mine of useful source material.

The Units and their leaders were:

- I Mathematics in Aeronautics—Dr. A. S. Otis, Yonkers-on-Hudson
- II Mathematics in Engineering—Professor J. W. Seeley, of Duke University
- III Arithmetic in the High School—Dr. Wm. Betz, Rochester, N. Y.
- IV Air Navigation—Professor F. G. Dressel, of Duke University

The registration fee was \$1 per unit, or \$3 for all four units. Room and board in the Duke University dormitories was \$2 a day for double room, and \$2.50 a day for a single room. The time covered by the institute from June 14 to 26 thus represented living expenses of \$26 or at the most \$32.50.

The 1944 Mathematics Institute followed the same structural plan as the 1943 Institute. This time there were five units, with which the teachers aligned themselves for research and special study. Again the Mathematics Laboratory and Library were in constant use. At the end of the sessions reports were made and published in a mimeographed volume "Mathematics Institute High Lights." A limited number of these are available to those interested in the Institute idea.

Unit I was under the leadership of Professor E. H. C. Hildebrandt of Northwestern University. "Visual Aids in the Study of Mathematics" was the subject, and was most thoroughly treated. Professor Hildebrandt showed a wealth of material including slides, models, charts, stereoscopic

views and pictures that can be used in teaching every branch of secondary school mathematics. The unit was warmly praised by all the teachers in attendance.

Unit II. Air and Sea Navigation, leader, Professor F. G. Dressel, Mathematics Department of Duke University. This unit covered such topics as "Dead Reckoning and Compass Fixing, Compass Errors and Simple Vector Problems, Radius of Action, and Aircraft to Carrier Problems, Instruments Used in Navigation." As part of the unit, there was an observation party in astronomy, conducted by Professor K. B. Patterson of Duke University. It was a most informing unit, and the lecture entitled "The Celestial Sphere and Time Diagrams" clarified concepts that in many minds had been hazy.

Unit III. Post-War Planning of Mathematics Programs. Leader, Miss Veryl Schult, Head of Mathematics Department, Division 1-9, D. C. Public Schools. Miss Schult organized the teachers of her unit into several committees, who devoted their time outside the lectures, to studying various phases of post-war curriculum planning. Some problems considered were:

1. Arithmetic in the Revised Mathematics Program.
2. The Laboratory Method.
3. How Can We Improve the Teaching of the Regular Sequential Course in Mathematics?
4. Where Can We Find the Best Problem Material?
5. The Value of Practical Applications in the Study of Mathematics.
6. Lessons from Observing the Training in the Armed Forces.

The findings are reflected briefly in the "Platform" adopted at the end of the session. The clarity of her exposition, and her skill in organizing her committees made Miss Schult a very valuable leader.

Unit IV. Consumer Mathematics. Leader, Professor W. S. Schlauch, of New York University.

This unit considered the mathematics useful to the consumer in budgeting the

family income, providing insurance protection, investing his savings, borrowing to meet emergencies, installment buying and borrowing, and amortizing his old age requirements.

The mathematics needed includes percentage, simple and compound interest, annuities, series, probability, elementary insurance mathematics, and some investment mathematics. The consensus of the Institute was that such mathematics could be taught adequately only after a course in preparation taken by the teacher, in most cases.

Unit V. Applications of Mathematics to Engineering. Leader, Professor W. J. Seeley, Chairman of Department of Electrical Engineering of Duke University.

The lectures of this unit were illustrated by apparatus, experiments, and the use of machines. The dependence of the engineer on mathematics, the measurement of speeds, the mathematics of air conditioning, and mathematical computation by means of electrical measuring instruments, were subjects of the lectures that gave the teachers in attendance first hand contact with apparatus, computation, and the absolute necessity for mathematical knowledge for the engineer. Professor Seeley is a remarkably clear and lucid expositor.

The leisure hours of the teachers were provided for by receptions, teas, dances and entertainments, in the home of Professor and Mrs. Rankin, in the University buildings, and on the campus.

A copy of the findings of the Institute was sent to the Commission on Post-War Plans of the National Council of Teachers of Mathematics. The platform adopted by the Institute contains so many excellent suggestions that we give it in full.

A PLATFORM FOR SECONDARY MATHEMATICS

We believe that in order to promote sound growth and development, both in teaching and the study of secondary mathematics, it is necessary to give heed to the following:

1. Mathematical libraries for schools should be provided with carefully selected books for teachers and pupils. They should include material on applications, historical information, philosophy of mathematics, methods books, periodicals, and puzzles.

2. Mathematical laboratories should be set up in different sections of the country, available to teachers and pupils throughout the school year including the summer.

3. Materials for the mathematics laboratory should be chosen with a view to make mathematics more meaningful. They should serve to develop mathematical principles, and to bring out fundamental relationships in the fields of business, science, social studies, industry, etc.

4. We favor a plan for continuously collecting up-to-date materials that are as practical as possible, and for distributing them widely for the benefit of mathematics teachers. We believe that these problems should be available to text book writers free from copyright.

5. We endorse appointment of a committee to study and evaluate the most worth while multi-sensory aids, with recommendations as to their grade-level placement, and their best use.

6. We strongly favor selecting a committee to evaluate curriculum studies, including some teachers of secondary mathematics, several school administrators, a teacher of mathematics on the college level, a physics teacher, an engineering teacher, a business man, an industrialist, and a social science teacher.

7. We are convinced of the great benefit accruing to the teacher who participates in the work of a mathematics institute. The benefits may be summarized as follows:

(a) Contact with a wide range of source materials in secondary mathematics, such as books, reports, instruments, models.

(b) Contact with recognized leaders in various fields.

(c) Participation by the attending teachers in groups working along the lines of the major topics of the institute.

(d) Opportunity for the exchange of teaching experiences.

8. We favor the promotion of such institutes throughout the Nation, and recommend setting up a committee to study the advisability of promoting mathematical institutes or workshops for teachers during the summer months.

9. We recommend that the National Council of Teachers of Mathematics select a committee to study the question of setting up a really adequate type of training for teachers of mathematics.

10. We recommend that colleges and university offer courses in statistics in order that teachers may be prepared to teach statistics on the secondary school level when feasible.

11. We recommend that courses in consumer mathematics be offered in high schools after teachers have been prepared in college or university for the teaching of this subject.

12. We believe that the Commission on Post-War Plans of the National Council of Teachers of Mathematics should be enlarged to include representatives from:

(a) College mathematics.

(b) Science.

(c) Social studies.

(d) Business.

(e) Industry.

(f) School administration.

In this way we are confident that the influence of the Commission can be greatly extended, and the prospects of a widespread adoption of their recommendations by school officials and administrators.

These twelve thought-provoking planks of the Platform should give rise to lively discussion and constructive action at the next meeting of the National Council of Teachers of Mathematics.

The enthusiasm shown by the participants, the excellent lectures, the real work done, and the delightful social events made the two weeks spent by the author in Durham, in early July, a source of delightful memories.

Some Thoughts on Placing the Decimal Point in Quotients

By CLAUDE H. BROWN

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TEACHERS of arithmetic, writers of arithmetic textbooks, and writers of books on the teaching of arithmetic agree that pupils should be taught to solve multiplication problems which involve decimals by multiplying as with integers and then pointing off a number of decimal places in the product equal to the sum of the number of decimal places in the multiplier and the multiplicand. There is also general recognition of the fact that division is the inverse of multiplication.

It is, therefore, quite surprising to learn that there exists now and that for a long time there has existed a decided difference of opinion among teachers and writers in the field of arithmetic concerning the proper method of locating the decimal point in the quotient in problems where division by decimals is involved.

As long ago as 1917, Monroe reported the results of investigations which showed that a variety of devices were used for locating the decimal point in quotients and that the same person even used different devices in different types of problems.¹

One of the methods for determining the location of the decimal point in a quotient which is widely taught at the present time is based on the fact that the value of the quotient is left unchanged if both dividend and divisor are multiplied by the same number. Because of this fact, therefore, any division problem may be transformed into one in which the divisor is an integer by multiplying dividend and divisor by some power of ten. Thus, if we have $1.296 \div .36$, we may multiply both dividend and divisor by 100 and have $129.6 \div 36$ without changing the value of the quotient. In

actual practice this multiplication is not performed but the changed position which the decimal point would occupy in dividend and divisor is indicated by a caret as follows:

$$.36 \wedge) 1.29 \wedge 6.$$

Pupils taught this method are instructed to locate the decimal point in the quotient directly above the caret in the dividend (Step I)

$$.36 \wedge) 1.29 \wedge 6$$

Step I

and then, after placing the decimal point, to divide as with integers (Step II). For the sake of brevity, this method will hereafter be termed the caret method.

The second widely used method for placing the decimal point in quotients is based on the fact that division is the inverse of multiplication. Teachers who prefer this method instruct their pupils to divide as with integers and then to point off in the quotient a number of decimal places equal to the number obtained by subtracting the number of decimal places in the divisor from the number of decimal places in the dividend. Thus, in the problem $1.296 \div .36 = 3.6$, the number of decimal places in the quotient is obtained by subtracting 2 (the number of places in the divisor) from 3 (the number of places in the dividend). This is, of course, merely the inverse of the procedure used in locating the decimal point in a product. This method will hereafter be termed the subtraction method.

Kendall and Mirick, as early as 1915, stated that the caret method was preferable because it was the method which was

$$\begin{array}{r} 3.6 \\ .36 \wedge) 1.19 \wedge 6 \\ \underline{1 \ 08} \\ 21 \ 6 \\ \underline{21 \ 6} \\ 0 \end{array}$$

Step II

¹ Monroe, W. S., "The Ability to Place the Decimal Point in Division," *Elementary School Journal*, 18: 287-293, December, 1917.

least likely to result in error² although they did not cite any evidence in support of this conclusion. Brown and Coffman, in a book published in 1924, advocated use of the subtraction method.³ However, in the same book, Brown and Coffman also advocated transforming problems involving repeated multiplication and division (commonly termed cancellation problems) into equivalent problems with integral divisors before attempting their solution.

The most recent books on the teaching of arithmetic still show this division of opinion concerning the proper procedure for division by decimals. Thus Morton suggests that pupils be led to formulate the rule: *The number of decimal places in the dividend minus the number in the divisor equals the number in the quotient.*⁴ On the other hand, Wheat,⁵ Taylor,⁶ and Wilson, Stone and Dalrymple⁷ prefer the caret method.

As has been pointed out, no such difference of opinion exists with respect to multiplication of decimals. In each of the books mentioned above, it is suggested that pupils be taught to point off a number of decimal places in the product equal to the sum of the number of places in multiplier and multiplicand. Since division is the inverse of multiplication, it would seem logical to teach the inverse of the rule, used in multiplication for locating the decimal point in division. This is the procedure advocated by Morton.

² Kendall, Calvin N., and Mirick, Geo. A., *How to Teach the Fundamental Subjects*, New York, Houghton Mifflin Co., 1915, pp. 177-179.

³ Brown, Joseph C., and Coffman, Lotus D., *The Teaching of Arithmetic*, Chicago, Row, Peterson and Co., 1924, p. 228.

⁴ Morton, R. L., *Teaching Arithmetic in the Elementary School*, Chicago, Silver Burdette and Co. 1938, Vol. II, p. 335.

⁵ Wheat, Harry G., *The Psychology and Teaching of Arithmetic*, New York, D. C. Heath and Co., 1937, pp. 413-417.

⁶ Taylor, E. H., *Arithmetic for Teacher-Training Classes*, New York, Henry Holt and Co., 1937, p. 176.

⁷ Wilson, Guy M., Stone, Mildred B., and Dalrymple, Charles L., *Teaching the New Arithmetic*, New York, The McGraw-Hill Book Co., 1939, p. 220.

What reason is given by the other writers for introducing an entirely different rule? In every case in which any justification of this practice is attempted, it is on the grounds that the caret method results in a higher degree of accuracy being obtained. Yet none of the works quoted present any objective evidence in support of this claim.

One obvious objection to the scheme of always transforming division problems into problems in which the divisor is an integer is the mechanical nature of the process. It is very little, if any, easier to impart a real understanding of the caret rule to a class than to impart an understanding of the subtraction rule. The caret method, however, may be taught as a trick, a short cut which the child may use without understanding why it works. If such is the case, despite the apparently greater degree of accuracy achieved in solving problems in schoolroom situations by the caret method, it is certain to prove inadequate when the pupil later attempts to use a slide rule or other computing machine, or attempts to solve problems involving repeated multiplication and division (cancellation problems) or problems in which it is inconvenient to write the quotient directly over the dividend—in short—any of the complex problems which are likely to arise in life situations. Furthermore, the caret method is accurate only if the figures in the quotient are placed directly over the proper digits in the dividend. This implies a degree of precision in writing figures which is difficult to obtain when the pupil is striving for speed or when his attention is centered on the conditions of his problem and not on the computation process. In other words, even if the caret method is more accurate in schoolroom situations, it does not follow that it will be equally accurate in out-of-school situations.

It is true that there are certain types of problems which are apt to cause difficulties when the subtraction method is used. Problems in which there are more decimal

places in the divisor than in the dividend may be cited as an example of these more difficult types. But this does not necessarily discredit the method. It is merely evidence of the fact that no method is, of itself, surety against error. Furthermore, the subtraction method, if mastered by the student, will serve to locate the decimal point properly in any situation involving division by decimals. This is not true of the caret method.

Probably the most significant factor in determining the choice of a method for locating the decimal point in division is the conception of the nature and educative function of arithmetic held by the individual making the choice. If arithmetic is to be taught merely for its utilitarian values, if the only educational purpose to be served by the teaching of arithmetic is that of developing in the individual the ability to compute with reasonable speed and accuracy, if an understanding of a given process is not essential, then use of a purely mechanical procedure like the caret method may be permissible.

If, however, as Morton suggests:

Arithmetic is conceived as a closely knit system of understandable ideas, principles, and processes, and an important test of arithmetical learning is an intelligent grasp upon number relations together with the ability to deal with arithmetical situations with proper comprehension of their mathematical significance. . . .*

then the more logical subtraction method seems to be indicated. That is, if the primary function of arithmetic teaching is development of the pupil's ability to compute, if no other factors need be considered, and if, as claimed, the caret method gives a higher degree of accuracy than the subtraction method, then the caret method might be preferable. If, however, the "chief purpose of arithmetic teaching is

development in the pupil of the ability to do quantitative thinking," if we are concerned with developing an understanding of the different processes and of their relationships to one another, then the subtraction method is more desirable.

It is possible that the most desirable approach to the problem of division by decimals is through a combination of the two methods. That is, the problem may be approached by first showing that if the divisor is an integer, then the number of decimal places in the quotient is exactly equal to the number in the dividend. Then division by decimals may be presented by the caret method. It is then necessary to go but one step farther. After the pupil has learned to use the caret method with confidence, it is only necessary to point out that this process is essentially one of subtracting the number of decimal places in the divisor from the number of places in the dividend. However, if the individual ever expects to use the slide rule, if he will ever be called upon to solve the problems involving repeated multiplication and division which arise in science courses, if it will ever be necessary for him to solve problems in which it is inconvenient to write the quotient directly over the dividend, then it is essential that this last step be taken.

Furthermore, any rule will soon be forgotten if merely memorized. If, on the other hand, the pupil formulates a rule based on his own insight into the relationships involved in a problem, he will not forget it so easily. Therefore, in teaching division by decimals, as in all arithmetic teaching, emphasis must be on understanding and not on mechanical techniques. Hence, as these facts indicate, the subtraction method of locating the decimal point in quotients is preferable and should therefore be taught either in addition to, or instead of, the caret method.

* Morton, R. L., *Teaching Arithmetic in the Elementary School*, Chicago, Silver Burdette and Co., 1939, Vol. III, p. 13.

◆ THE ART OF TEACHING ◆

Adjusting Low Scores

By G. W. GROSSMAN

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It is a common experience of teachers to find after marking the test papers of a class, that all the marks are very low. Sometimes it is evident that the fault lies in the fact that the teacher has made the test too difficult, or in some other factor which can not be justly attributed to the poor work of the students. When the teacher realizes this she desires to adjust these marks to equivalent scores which she feels are just, and which she can use in combination with other marks in making up a final grade for the student.

Several methods of adjusting these low scores are being used. Some teachers simply add a certain amount to each score. This is not satisfactory for the higher scores because they become too high. Some have added a fractional part of the difference between the low score and one hundred. This has some merit but it involves considerable work and the low scores are given a disproportionate advantage.

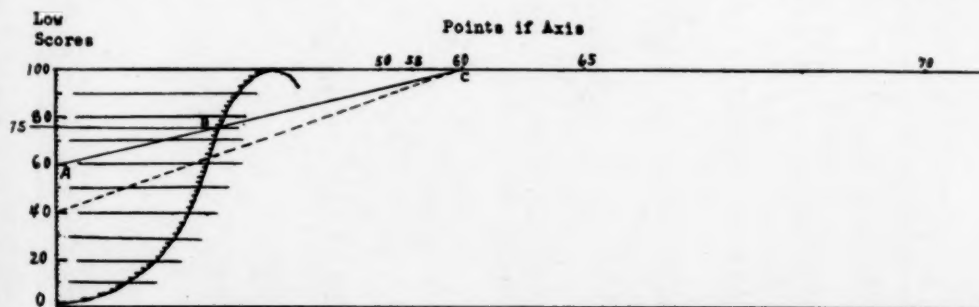
A consideration of these conditions has developed a procedure that seems to be

both satisfactory in results and convenient to use.

A graph is used to secure the corrected scores which are to replace the low scores on the test. By its use corrected scores are obtained which are never more than one hundred, and which are not unduly increased. Scores obtained from this graph may be arranged in a table and are readily available for the adjustment of any low score without further calculations. A study of the following graph, which is based on a three sigma length of the normal curve with a half width equal to 90% of its height, will reveal how the table is obtained.

After the graph is drawn the "Points of Axis" are determined by drawing a straight line from the "Low Score" which is deemed high enough for passing, through the corrected score of seventy-five to the top of the graph.

When the several points of axis are determined then the only judgment the teacher needs to make on certain low test



GRAPH FOR DETERMINING ADJUSTED SCORES FROM LOW SCORES

Note: When a score of 60 is judged equivalent to passing then a score of 40 would be adjusted to 62 as seen on the normal curve.

scores is to decide what score she would consider equal to a passing mark, and use that score as the point of axis.

After the point of axis is chosen a line from that point to any low score will pass through its appropriate corrected score.

It is apparent that the ratio of the length of the line *AB* to *BC* is constantly changing so that as the low score approaches 100 the corrected score also approaches 100 and reaches that point simultaneous with it.

It is also apparent that as a low score approaches zero the corrected score is reduced so as not to give a disproportionately large value. There seems to be no

mathematical proof that the corrected scores obtained from the use of the normal curve in this graph are the correct ones to use. However, they do have a factual basis and seem to give acceptable results.

By use of this graph the following table was made which not only gives the corrected scores for any test whose passing score is 50, 55, 60, 65, or 70, but it also gives the corrected letter grade when scores are given in letters.

To use the table simply find the low score in the first column and then the corrected score will be found to the right in the column headed by the score considered equivalent to passing.

CONVERSION TABLE FOR LOW SCORES

	Low score	Score equivalent to passing				
		70	65	60	55	50
A	100	100	100	100	100	100
	99	99	99	99	100	100
	98	98	99	99	99	100
	97	98	98	98	99	99
	96	97	98	98	98	99
B	95	97	97	97	98	99
	94	96	97	97	97	98
	93	95	96	96	97	98
	92	94	95	95	96	98
	91	93	94	95	96	97
C	90	92	93	94	95	97
	89	92	93	94	95	96
	88	91	92	93	94	96
	87	90	92	93	94	95
	86	89	91	92	93	95
D	85	88	90	92	93	94
	84	87	89	91	92	94
	83	86	88	90	92	93
	82	85	88	89	91	93
	81	84	87	89	91	92
F	80	83	86	88	90	92
	79	83	86	88	90	91
	78	82	85	87	89	91
	77	81	84	86	89	90
	76	80	84	86	88	90
	75	80	83	85	88	89
	74	79	82	84	87	89
	73	70	81	84	86	88
	72	76	80	83	86	88
	71	75	79	82	85	87
	70	75	78	81	84	87
	69	74	78	81	84	86
	68	73	77	80	83	86
	67	73	77	80	82	85
	66	72	76	79	82	85
	65	72	76	79	81	84
	64	71	75	78	80	84
	63	70	74	77	80	83
	62	69	74	77	79	82
	61	69	73	76	78	81

	Low score	Score equivalent to passing				
		70	65	60	55	50
	60	68	72	75	77	80
	59	67	72	75	77	80
	58	67	71	74	76	79
	57	66	71	74	76	79
	56	65	70	73	75	78
	55	64	69	73	75	78
	54	64	69	72	74	77
	53	63	68	72	74	77
	52	62	67	71	73	76
	51	61	66	70	73	76
	50	60	65	69	72	75
	49	60	65	69	72	75
	48	59	64	68	71	74
	47	58	63	67	71	74
	46	57	62	67	70	73
	45	56	61	66	69	73
	44	56	60	65	68	72
	43	55	59	64	67	71
	42	54	58	63	67	71
	41	53	57	62	66	70
	40	52	56	61	65	69
	39	51	56	61	65	69
	38	50	55	60	64	68
	37	49	54	59	64	68
	36	48	53	58	63	67
	35	47	52	58	63	67
	34	46	51	57	62	66
	33	45	50	56	62	65
	32	44	49	55	61	65
	31	43	48	54	60	64
	30	42	48	53	59	63
	29	41	47	52	58	62
	28	40	46	52	58	62
	27	39	45	51	57	61
	26	38	45	51	57	61
	25	37	44	50	56	60
	24	36	43	49	56	59
	23	35	42	48	55	59
	22	34	41	47	54	57
	21	34	40	46	53	56

EDITORIAL

IN A RECENT editorial in *The National Mathematics Magazine* Professor S. T. Sanders said:

Many teachers are nervously concerned over what may be the post-war status of school mathematics. The enormous expansion of the technical applications of the science under pressure of war has brought about a world-wide strengthening of mathematics in the school curriculum. Can this current academic primacy of mathematics be made permanent? Such is the question raised by those keenly mindful of the scant attention paid to this subject by the less recent curriculum makers.

A careful study of the matter should not discount the fact that in respect to mathematics, the war has served only to bring about greatly multiplied uses of mathematics a large proportion of which were already in existence. For, even in pre-war times there had been for many years a steadily growing public emphasis upon applied mathematics, rather than upon the logical or cultural aspects of the science.

In the light of this definite trend, a trend not rooted in any war, it could well be that the post-war school effort should first be directed to discovering the mathematical aids or needs of all the major peace-time industrial enterprises. Cooperative programs initiated between industry and the schools would then have sounder foundations. Who shall say that the cultures of mathematics would be impaired by being stemmed in its utilities?¹

That great interest is being manifested in what place mathematics is to have in the schools after the war is evidenced by the many discussions that one hears, the various articles now appearing in current magazines and editorial comment like that above. Moreover, as the readers of *THE MATHEMATICS TEACHER* know, The National Council of Teachers of Mathematics has appointed a *Commission on Post-War Plans* in mathematics the first report of which appeared in *THE MATHEMATICS TEACHER* for May 1944. A second and more inclusive report of progress of this

commission will appear in the May 1945 issue of this journal.

The problem about which we are concerned here is that we do not have agreement in all quarters as to what should be done. We cannot take the space to survey all of the recent opinions and articles, but a few typical ones will show how the wind is blowing.

After discussing the high place which mathematics once held in the schools and the poor results in mathematical education shown by recent Army reports, Professor Harold L. Dorwart says:

At this point, it may be asked why so many of our young people ceased to study mathematics some years ago. Many mathematics teachers say that it is all the fault of the educationists with their half-baked theories of the nontransfer of training and of the removal of everything difficult from education in order to prevent harmful personality development. The retort of the educationists is that mathematics teachers are just a lot of sadistic drill-masters who do more harm than good anyway. I will pass over this charge and countercharge in favor of what may, it is hoped, be a somewhat more helpful point of view.

But first, let us face the fact that there have been, and still are, both poor and poorly prepared teachers of mathematics, and that they have repelled many good students. Many high-school principals must be forced to give up the idea that anyone who possesses the credits in methods-of-education courses specified by law is thereby qualified to teach algebra or geometry. Also, college presidents and deans should investigate carefully the personalities of the instructors assigned to elementary courses. Even if the instructor is a recognized authority in his field, the conceited, arrogant, show-off type (fortunately few in number) should be used elsewhere than in elementary courses.

I now propose the thesis that, once the requirements were withdrawn, students ceased to study mathematics principally because they did not recognize the fundamental role that it plays in modern civilization, or that by omitting the study of mathematics they were thereby imposing large restrictions on their future choice of profession or employment. In short, they did not recognize and usually have not been told

¹ Sanders, S. T., "Post-War Planning in Mathematics," *National Mathematics Magazine*, October 1943, page 2.

that, in addition to serving, mathematics is a queen in her own right, a queen who will richly reward her followers, but only if they follow her diligently from their youth through a long period of time, even when the going is tough and when the path ahead is not always crystal clear.²

Thus far the comments and articles referred to have been favorable to mathematics. However, we must present a typical point of view which raises questions as to whether mathematics is as important in the secondary school as some people think. Professor Frank N. Freeman, Dean of the School of Education at the University of California recently said:

One of the outcomes of the war, in the opinion of many officers of the Army and Navy and of many observant laymen, is the revelation of gross inadequacy in the teaching of mathematics. The experience of the armed forces and of industry is supposed to have shown that vastly larger numbers of students should study mathematics and that they should study more mathematics—up to and including trigonometry. We may look, therefore for a campaign after the war, to require the study of mathematics through trigonometry by a large share of high-school students. If such a campaign is launched, it will grow out of an idea that is about as sound as the belief that the psychological tests given in World War I showed 40 per cent of the male population to be morons.

The reasons that no such conclusion follows from the experience of the Army and Navy, not to speak of industry, are, first, that the demands of the services in wartime are no reflection of their demands in times of peace, and, second, that the number of men who need straight mathematics to perform duties required of them is much fewer than has been implied by the published statements. To say that the schools should, in peacetime, give all the preparatory training that may be needed in time of war is like saying that industry must be kept in continual readiness to produce 10,000 planes a month. Again, the number of men who acquire techniques in the Army or Navy which are based on mathematics is vastly greater than the number who are required to understand the principles of mathematics and their application. The same is true of industry. The implied statement that all the men in the armed forces and industry who perform technical operations require a knowledge of higher mathematics for such performance is the wildest exaggeration. The war does not teach that mathematics through trigonometry is a practical necessity for

a large proportion of men, let alone women. The question of the kind and amount of mathematics which is good for the average person and should be an element in general education still remains open.³

In the same article Dean Freeman presents a series of propositions which he thinks may be taken as a platform for the reorganization of mathematics in general education as follows:

1. Only that mathematics is important for general education which the individual will use.

2. The individual will use only those mathematical ideas and operations which he has learned by or in use.

3. The individual will actually use only those processes that he has mastered and made thoroughly familiar to himself.

4. Understanding is desirable, but it comes best through familiar use first, and formal explanation afterward.

5. Mathematics may properly be thought of as a language—that is as a particular set or particular sets of symbols which represent special aspects of reality.

6. Mathematics is, or contains, a form of language which formulates and defines the quantitative aspect of experience and, therefore, stimulates and largely creates quantitative ideas and forms of thought.

7. To have meaning in the thinking of the child or of the ordinary person in general, the use of the mathematical symbols and operations should be developed in intimate and continual association with the real world of experience.

8. The mathematics for the million is that which gives clearer and more effective ways of thinking about the real world of experience because it has been developed out of this world of experience.⁴

We now come to the main point of our problem, namely, how can such apparently diverging points of view be reconciled in the post-war years?

President James Bryant Conant of Harvard University has recently given an excellent suggestion. In discussing the reasons why the lay critics of secondary education talk as they do, President Conant says:

I am almost tempted to generalize that the more educated the person, the less his knowledge

² Freeman, Frank N. "Teaching Mathematics for the Million." *California Journal of Secondary Education*. May 1944, pp. 246-254. See also Caswell, H. L., "Progressive Education Principles Used in the War Effort." *Teachers College Record*. March 1944, pp. 386-397.

⁴ *Ibid.*, pp. 246-254.

² Dorwart, Harold L., "Mathematics—Queen and Handmaiden." *School and Society*. October 14, 1944, pp. 241-243.

of secondary school education. Certainly the lack of knowledge among the professors of arts and sciences in our colleges and universities is proverbial. And with lack of information goes lack of understanding and lack of sympathy. As a result, on more than one campus we have almost a state of civil war between those who profess a knowledge of education and those who profess a knowledge of subjects which constitute a modern educational curriculum.

This academic war has been in a sense inevitable, as I propose to show by a brief resume of history, but to my mind an armistice has been for some years overdue. And it is for such an armistice that I should like to put in a good word this afternoon (and I might remark parenthetically that it takes two to make an armistice quite as much as to make a quarrel). My belief in the need for the cessation of hostilities comes not only from my general tendency to favor pacific methods of handling academic controversy, but also because I am really worried about the present lay reaction to educational matters. I am distressed by both the vehemence and the ignorance with which views about education are expressed publicly and privately by many prominent people. Now we can hardly expect the public to have a very clear understanding about educational problems when education is a house warring against itself.

Hence my plea this afternoon for a "cease firing" order.¹

This is the attitude that we have been taking for some time. When teachers of secondary mathematics, college mathematics, supervisors, administrators and general educationists sit down around the table to discuss what for all of them should be a common problem, then we can hope for some practical solution of what shall constitute general education in the post-war years. A good preparation for such a procedure for all those who are interested in what the content of post-war mathematics should be is a most careful reading of the entire articles referred to above as well as of some which we have not had space to mention.

W. D. R.

¹ Conant, James Bryant, "A Truce Among Educators," *Teachers College Record*. December 1944, pp. 157-163.

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The American Mathematical Monthly

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1. Cairns, S. S., "Deformations of Plane Rectilinear Complexes," pp. 247-252.
2. Cole, R. H., "Associated Frequency Distributions in Biometry," pp. 252-261.
3. Segre, B., and Mahler, K., "On the Densest Packing of Circles," pp. 261-270.
4. Wilcox, L. R., "An Application of a Theorem of Sylvester," pp. 270-271.
5. Larsen, H. D., "A Note on Scales of Notation," pp. 274-275.
6. Lovitt, W. V., "Reciprocal Equations," pp. 276-277.
7. Dubisch, Roy, "The Roots of a Cayley Number," p. 278.
8. "War Information: Salaries and Teaching Loads in College Training Programs; Educational Credit for Military Experience; Future of the Navy V-12 Program; Notes on the Training Programs; Selective Service Regulations Concerning Students," pp. 298-303.

Bulletin of the Kansas Association of Teachers of Mathematics

December 1944, vol. 19, no. 2.

1. Albright, Penrose S., "Trends of the Present Air Age on Mathematics," pp. 19-21.
2. "The Round Table Meetings," pp. 21-23.
3. "Mathematics in the Rehabilitation Program," pp. 23-24.
4. Shirk, J. A. G., "Needs of Post-War Teaching of Mathematics," pp. 24-25.
5. Highbaugh, Swan, "Outline of Shop Mathematics as Taught in Coffeyville Trade School," pp. 25-26.
6. Read, Cecil B., "Arthur J. Hoare—a Tribute," pp. 26-27.
7. Hall, Lucy, "M. Bird Weimar—a Tribute," pp. 27-28.
8. Dorwart, Harold L., "Mathematics—Queen and Handmaiden," pp. 28-30.
9. McKown, John, "A Derivation of Heron's Formula," pp. 30-31.
10. "A Platform for Secondary Mathematics," pp. 31-32.

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November 1944, vol. 19, no. 2.

1. Sanders, S. T., "The Lure of the Infinite," p. 54.
2. Eves, Howard, "Skew Curves Setting Up a Null System in Space," pp. 55-61.

3. Baker, G. A., "F-Values for Samples of Four and Four Drawn from Populations Which are the Sum of Two Normal Populations," pp. 62-63.
4. Miller, G. A., "A Ninth Lesson in the History of Mathematics," pp. 64-72.
5. Hassler, J. O., "A Method of Measuring Effectiveness in Teaching College Mathematics," pp. 73-77.
6. Charosh, Mannis, "Unifying Elementary Mathematics by Means of Fundamental Concepts," pp. 78-90.
7. Ballard, Ruth Mason, "Notes from a Freshman Class Room," pp. 91-92.

School Science and Mathematics

December 1944, vol. 44, no. 9.

1. Hunt, Dannel H., "Mathematics Tuned to the Times," pp. 789-792.
2. Blank, Laura, "Principles of Solid Geometry Basic to Navigation," pp. 813-818.
3. Loomis, Hiram B., "Pandiagonal Magic Squares and their Relatives," pp. 831-838.
4. Breslich, E. R., "David Eugene Smith, 1860-1944," pp. 838-839.
5. Nyberg, Joseph A., "Notes from a Mathematics Classroom," pp. 854-857.

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1. Backus, A. D., "Trisecting That Angle," *Industrial Arts and Vocational Education*, 33: 390, November, 1944.
2. Barclay, E., "Christmas Arithmetic Applied to a Community Project," *The Grade Teacher*, 62: 48+, December, 1944.
3. Bowers, E., "How Problem Material in Mathematics Textbooks Has Changed During Several War Periods," *Baltimore Bulletin of Education*, 22: 25-28, September, 1944.
4. Brownell, W. A., "Rate, Accuracy and Process in Learning," *Journal of Educational Psychology*, 35: 321-337, September, 1944.
5. Manheimer, W. P., "Mathematics and the Core Curriculum," *High Points*, 26: 71-73, October, 1944.
6. Rosenberg, R. R., "Diagnostic Tests in Business Mathematics," *Business Education World*, 25: 157-159, November, 1944.
7. Sauble, I., "Teaching Fractions," *The Instructor*, 54: 29-30+, November, 1944.
8. "School Certificate Mathematics," *Nature*, 154: 234-235, August 19, 1944.

NEWS NOTES

A meeting of the Mathematics Section of the Winnipeg Teachers Convention was held in the Kelvin Technical High School, Friday, November 24. D. McLeod, Daniel McIntyre, Collegiate, presided. The secretary was Miss C. Wood, General Wolfe School.

Papers were read by J. G. Johannsson, also of the Collegiate, and by H. E. Riter of the Provincial Normal School. The topics were (1) "The Grade XI Mathematics Examination" and (2) "The Development of Reflective Thinking through Problem Solving."

The first speaker advocated the elimination of useless and "dead wood" material from the algebra course. He dealt with questions selected from examination papers and showed how many improvements could be made.

Mr. Riter gave a lengthy and interesting paper on reflective thinking as applied to mathematical and every day situations. Many questions in mathematics are done by rule which should really be thought out. In algebraic problems analytic dependence might be used to great advantage, while in geometry the use of converses, inverses, contrapositives and syllogistic reasoning should be emphasized. He referred his audience to (1) The Thirteenth Year Book of the National Council of Teachers of Mathematics, which deals with "The Nature of Proof" and (2) several articles in THE MATHEMATICS TEACHER. A vote of thanks was passed. D. McLeod.

The third meeting of the Men's Mathematics Club of Chicago and The Metropolitan Area on December 15 was devoted to an open forum on the topics "I Say What I Think."

Discussions of special teaching techniques, reviews of books or magazine articles, recreational mathematics, personal reminiscences of great teachers, etc., were in order. Every member considered himself a potential speaker.

The Annual Meeting of the Mathematics Section of the Arkansas Education Association was held at the Senior High School, Little Rock, Arkansas on November 14, 1944. Lt. Col. Jay Dykhouse, Chief, Pre-Induction Training Branch Military Training Division, of Dallas, Texas was the guest speaker. Dr. Harry I. Lane of Hendrix College, Conway, is President and Mr. W. E. Shelton, North Little Rock is secretary. The newly elected officers for the coming year are Mrs. Joe Goetz of North Little Rock, President, and (Miss) Elizabeth McHenry of Little Rock, Secretary.

Little, Brown & Company announces that it has recently decided to discontinue publication of school and college textbooks, including Atlantic Monthly Press textbooks, and has sold to D. C. Heath and Company of Boston its active textbook list.

Plans are under discussion between D. C. Heath and Company, Little, Brown & Company, and the Atlantic Monthly Press whereby

textbooks issued by D. C. Heath and Company, which have trade edition possibilities, will be handled in trade editions by Little, Brown & Company, trade books or manuscripts originating at Little, Brown & Company and the Atlantic Monthly Press, which have textbook possibilities, will be handled in textbook editions by D. C. Heath and Company.

The University of Chicago presented a series of public lectures on "Mathematics and the Imagination" at 4:30 P.M. in Room 122, Social Science Research Building, 1126 East Fifty-ninth Street.

January 16—"Imagination in Mathematics." EDWARD KASNER, *Professor of Mathematics, Columbia University*

January 17—"Imagination in Mathematics"—Continued. EDWARD KASNER, *Professor of Mathematics, Columbia University*

January 18—"Mathematics and the Visual Arts." WILLIAM IVINS, JR., *Counselor, Metropolitan Museum of Art, New York*

January 23—"Mathematics and the Visual Arts"—Continued. WILLIAM IVINS, JR., *Counselor, Metropolitan Museum of Art, New York*

January 25—"Mathematics and the Laws of Thought." MILTON SINGER, *College of the University of Chicago*

January 30—"On Poetry and Mathematics." SCOTT BUCHANAN, *Dean, St. John's College, Annapolis*

February 1—"Mathematical Mysticism and Mathematical Natural Science." ERNST CASSIRER, *Professor of Philosophy, Columbia University*

February 6—"Music and Mathematics." MANFRED F. BUKOFZER, *Associate Professor of Music, University of California*

February 8—"The Biological Basis of Imagination." RALPH WALDO GERARD, *Professor of Physiology, University of Chicago*

The Fourth Meeting of the Men's Mathematics Club of Chicago and the Metropolitan Area was held on Jan. 19th. Professor Chester Willard of Northwestern University spoke on "Education of Veterans" and Professor G. E. Moore of the University of Illinois spoke on "Mathematics in the Post-War World."

The following joint program of the Science and Mathematics Teachers of the Middle States Association of Colleges and Secondary Schools and Affiliated Associations was given at the Hotel New Yorker in New York City on Saturday, November 25th:

ASSOCIATION OF SCIENCE TEACHERS OF THE MIDDLE STATES

President—IVOR GRIFFITH, Philadelphia College of Pharmacy and Science

Vice-President—ELMER FIELD, South Philadelphia Girls High School

Success is not uniform, as we might expect. But it is to be noted that, in general, the instructors who use a cultural textbook for the cultural terminal student are not dissatisfied. Furthermore, two-thirds of the students using a cultural textbook indicated that their interest in mathematics had changed and nearly sixty per cent of these students indicated that general mathematics was more interesting than the traditional mathematics of high school. Some people are making friends for mathematics!

PHILIP L. SMITH

Elementary Statistics and Applications. By James G. Smith and Scheson J. Duncan. McGraw-Hill. 1944. vii + 720 pages. Price, \$4.00.

This new book is a text for a first course in general statistics. It gives methods of summarization and comparison, frequency distribution, the normal curve, simple linear and nonlinear correlation, multiple and partial correlation, time series analysis and forecasting. The mathematical and theoretical approach is emphasized and considerable attention is given to the fundamentals of the theory of statistics and to practical applications. The book also includes recent advances in the field.

The authors are teachers of statistics of years of experience and the book is the final compilation of material that was used in mimeograph form in their classes and continuously revised over the years.

W. D. R.

Industrial Series. by John W. Breneman. McGraw-Hill. 1944. Second Edition. xii + 221 pp. Price, \$1.75.

This book was prepared under the direction of the Division of Engineering of the Pennsylvania State College. The first edition was found to be helpful in courses given under the Engineering, Science, and Management War Training program. The second edition differs mainly from the first in changes due to suggestions from teachers who used the latter and in the addition of a large number of problems from a variety of sources.

On the whole the work is rather elementary as far as basic principles are concerned and is easily within the grasp of ninth and tenth grade pupils. Some of the applications are different from those found in conventional texts, but they are not too difficult to solve.

W. D. R.

Nautical Mathematics and Navigation. By S. A. Walling, J. C. Hill and C. J. Rees. Cambridge Press. 1944. ix + 221 pp. Price, \$2.00.

According to the authors this book was designed to help those who have become "rusty" in their knowledge of the elementary principles of mathematics and their nautical application. It is also intended to stimulate new interest in those who found their earlier work in mathematics hard or distasteful. The main purpose of the book is to enable the reader to apply the fundamental rules of the subject to the many interesting and important problems of seamanship.

W. D. R.

Today's Geometry. By David Reichgott and Lee R. Spiller. Prentice-Hall. 1944. xvi + 400 pp. Price, \$1.96.

This book is a revised edition of an earlier one. It keeps formal demonstration at a minimum and is a logical development and enlargement of the original text with special emphasis on some of the newer trends. New material dealing with global maps (Chapter V) and with vectors and air navigation (Chapter VI) are made an integral part of the book.

There is also a "refresher" section intended to restore skills in arithmetic and algebra. Numerous illustrations help to make the content material more interesting and helpful.

W. D. R.

Pre-Service Course in Machine Science. By Samuel H. Lebowitz. John Wiley and Sons. 1943. vi + 440 pp. Price, \$2.50.

This book was prepared at the request of the War Department and the U. S. Office of Education in conformance with the official pre-induction training course outline No. PIT102. It gives that subject matter required in the course on Fundamentals of Machines. The text is divided into 14 chapters conforming in content to the corresponding sections of the syllabus. It is intended to cover the work of a one-semester course.

Students who have the interest and the will to master the contents of this book will find themselves equipped with a great deal of basic knowledge and ability to analyze and understand various types of mechanisms.

W. D. R.